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Second-order accelerator of investment: The case of discrete time

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ABSTRACT

This paper presents a discrete time version of Hillinger's (1992,2005) second order accelerator model that investigates the dynamic behavior of capital, for pedagogical purposes. Such a version is put forward as a means of improving student acquaintance with the analysis of investment cycles -defined as quasi-periodic cyclic movements of capital- and with the convergence towards the steady-state when capital is subjected to trigonometric oscillations. In addition, we extend the analysis, introducing the exogenous interest rate on loans in the behavioral equation of investors. It is inferred that the introduction of this credit term results in a lower equilibrium level of capital.

1. Introduction

In order to categorize business cycle theories, different criteria can be used. A common criterion concerns the 'exogeneity' or 'endogeneity' of the cycles. More specifically, if the fluctuations are caused by external shocks, the cycle is 'exogenous'. These shocks may be completely random and non-cyclical. However, most Keynesians follow an alternative approach. They claim that although external shocks may exist and influence the economic activity, fluctuations would occur even in the absence of shocks. From this perspective, business cycles are endogenous. The reader can refer to the books of Zarnovitz (1992) or Susho (2006) to see the history and the theories of business cycles.

The theories that examine the goods' market give major emphasis on the determination of investment (Skott, 2012). Taking into consideration that investment constitutes a main part of total expenditure, the existence of high investment results in high aggregate demand and output. Moreover, this high level of output is followed by an increase in capital utilization and profitability that normally leads to an increase in investment and output in the next period. However, the convergence of the economic system towards its long-run equilibrium depends on the sensitivity of investment decisions in

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capital utilization and profitability. In particular, if this sensitivity is high, the result is a stable time path for output. On the other hand, a low level of sensitivity leads to divergence of output from its long-run equilibrium.

A primal work on business cycles modeling was done by Tinbergen (1930). His model, a model of industrial investment cycle, introduced the concept of time to build. This was achieved assuming the existence of a time lag between the inception and the completion of an investment project in shipping industry. The combination of this discrete time relation with other continuous time relations in this model led to a system of mixed difference-differential equation. Thus, the complexity of the solution of such model may provide an explanation for the limited adherence of this approach.

The argument behind the *General Theory of Employment, Interest and Money* (Keynes, 1936) was the role of the Great Depression as the cause of permanent structural changes in industrialized nations' economies. Among the factors of market disequilibrium that Keynes (1936) indicated, the price rigidity is that usually cited in the theory of investment cycles. In addition, the determination of these investment cycles was based on the relationship between the existing capital stock and the demand for output. This was the intuition behind the 'acceleration principle'. Then, the interest shifted to the inventory fluctuations and the inventory cycle, while the accelerator-multiplier mechanism became the centerpiece of Keynesian macroeconomic models. Samuelson (1939) was the first to form an accelerator-multiplier model. His analysis was based on a unit lag in both the consumption and the investment equations. In order to explain the inventory cycle, Metzler (1941) also applied this interaction.

Westerhoff (2006) extended the previous model, introducing a non-linear mix of extrapolative and regressive expectation formation mechanism. In this way the model is useful for the implementation of economic policy as well. Using the Samuelson's analysis as an inception, Jones (2012) showed the usefulness of competition trade theory in the general equilibrium theory. Moreover, Bond and Driskill (2009) studied the concept of dynamic stability in a trade model with three steady-state equilibria. An extension of the Samuelson multiplier-model was also presented by Dassios et al. (2014) who incorporated delayed variables in the initial model. Their analysis achieves to interpret the origination of stable business cycles if realistic and stochastic values of both the multiplier and the accelerator are concerned. Following a similar approach, Dassios and Zimbidis (2014) studied the stability of the national economy in the case of an interaction among different countries. In the same manner, Dassios and Kalogeropoulos (2014) attempted to explain the existence of stable business cycles in the context of a foreign trade model of three countries. In addition, Puu et al. (2004) proposed a Hickian type (Hicks, 1950) model that concentrates on the "floor" of income and omits the "ceiling". In particular, the "floor" is combined with the depreciation of capital stock. At the same time, they introduced a new dynamic method that is based on the relative growth rates. This model interprets the existence of business cycles with increasing amplitude.

Generally, the theoretical models in this field generate investment cycles and suggest that the roles of consumption and investment are of the same importance. This suggestion is not confirmed by empirical observations which imply that investment is more sensitive to the business cycle than the consumption. Hillinger (1992) developed a model in continuous time, explaining the central role of investment and inventory decision. As we will see, the fact that short run adjustments occur in quantities, not in prices, is a basic assumption of the theory of investment cycles and marks a great divide to the neoclassical paradigm. Hillinger et al. (1992) derived the second order accelerator for both fixed investments and inventories, regarding the microeconomic firm behavior. Hillinger and Weser (1988) and Weser (1992) used the second order accelerator model to discuss the aggregation problem that arises in business cycle theory. In the same manner, Woitek (1995) and Barnett et al. (1996) examined the business cycle stylized facts following an empirical approach.

It is generally accepted that any model is simply a formal representation and approximation designed to capture the essential of some economic process for the purpose of investigating certain economic behavior and its policy implications. Theoretical models formalize some aspect of economic behavior under a set of assumptions, which are usually chosen so that the model allows an analytical solution. The properties of the model can then be derived and their implications studied. Dynamic models directly incorporate time into their structure. This is usually done in economic modeling by using mathematical systems of difference or differential equations. Our contribution is to develop a version of the SOA (Second Order Acceleration) equation model of Hillinger (1992, 2005) that can be taught to undergraduate and median postgraduate students. Our opinion is that it is more convenient for students to present a model in discrete time given that the initial model is in continuous time (see e.g., Friedberg, 2010).

The paper is structured as follows. Firstly, we discuss the theoretical model (Section 2). Then, the solution of the model (Section 3) and the stability conditions of the system (Section 4) are presented. Section 5 shows a graphical representation of the roots. Section 6 introduces the exogenous interest rate on loans into the fundamental behavioral equation of investment. Section 7 concludes this educational paper with a summary of our findings.

2. The Model

We derive the second order accelerator of net investment using the standard flexible accelerator (Hillinger, 1992). In discrete time, our model is described by the following set of difference equations:

$$I_t - I_{t-1} = c(I_t^* - I_{t-1}), \quad 0 < c < 1 \tag{1}$$

$$I_t^* = b(K_t^* - K_{t-1}), \quad b > 0 \tag{2}$$