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An argument for the use of the square-root functional form in teaching undergraduate microeconomics☆

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ABSTRACT

The present paper has two objectives, both of which are purely pedagogical in nature. The first is to propose a competitor to the conceptual "workhorse" of economics, the Cobb–Douglas functional form. Our competitor is a special case of the square-root functional form. The second objective is to employ our special case as the foundation for a much-needed overview or roadmap to a large expanse of producer and consumer theory.

1. Introduction

Over a generation ago, Siegfried et al. (1991, page 202) aptly observed that: "the economics major is a helix—plowing the same ground repeatedly at progressively greater depth." One take (but not their take) on the meaning of the metaphor, "the economics major as a helix", is that the economics major (as a program of study) embodies a three-stage progression of helix-like transitions: (a) Stage 1: the almost exclusive-use of graphical methods in first-year economics courses, (b) Stage 2: the mixed use of graphical, numerical, and analytical methods in intermediate-level economics, and finally (c) Stage 3: the almost-exclusive use of analytical methods in the advanced economics courses.

At any one time, or in any one setting, this three-stage progression has concreteness only in the context of a particular functional form, of which there are many.[1,2] Of the lot, the functional form best suited to a broad range of applications is the

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[1] Here, we remind the reader of the distinction between a particular functional form (or a particular function) and an arbitrary functional form (or an arbitrary function). Likewise, we remind that the family of particular functions is comprised of the class of polynomial, rational, algebraic, and non-algebraic functions [Chiang and Wainwright (2005, Chapter 2)].

[2] For a catalogue of particular functional forms commonly used in microeconomics, see Berck et al. (2005), Chung (1994), or Griffin et al. (1987).

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Cobb–Douglas functional form. As a consequence, the Cobb–Douglas functional form has been dubbed the conceptual "workhorse" of economics [e.g., see Caselli and Feyrer (2007, page 553), Cornes (1992, page 49), and Klump et al. (2007, page 184)].

The present paper proposes that we educators might consider a second contender for the title of conceptual workhorse, at least in the classroom setting. The form proposed here is a special case of the square-root functional form. And while some may argue that our proposed functional form comes at a cost, we will argue below that its cost is more than offset by its principal redeeming property, its analytical simplicity.

This paper is organized as follows. In Section 2, we define the general square-root functional form, we define our special case of it, and we provide initial comments on its properties. In Section 3, we put our functional form to work by presenting (what we see as) a much-needed global overview to large expanses of both producer and consumer theory, an overview which is set in the context of a two-good world. In Section 4, we compare and contrast the properties of the two conceptual workhorses under consideration here: the established (viz., the Cobb–Douglas functional form) and our late entry (viz., the square-root functional form). Summary comments are offered in Section 5.

2. The square-root functional form defined

A few years before the appearance of the paper by Siegfried et al. (1991), Griffin et al. (1987) published a paper entitled "Selecting functional form in production function analysis." In it, they enumerated, and reviewed the properties of, twenty functional forms. One of these twenty is the square-root production function, which takes the general form of:

$$y = \alpha + \sum_{i=1}^{n} \beta_i \sqrt{x_i} + \sum_{i=1}^{n} \sum_{j=1}^{n} \delta_{ij} \sqrt{x_i} \sqrt{x_j} \tag{1}$$

where y denotes output, where x_i and x_j denote the volume of ith and the jth factor inputs, where $i = 1, n$ and $j = 1, n$, and where $n \geq 2$.

Eq. (1) leads us next to the following four remarks:

Remark 1—The Origins and Current Status of Eq. (1): One might note that:

- The origins of Eq. (1) are Heady (1957), Heady and Pesek (1960), Heady and Dillon (1961), which for our convenience we refer to collectively as the research of Heady & Co.[3]
- It appears that the square-root functional form [viz., Eq. (1)] never went mainstream, with the exception of the contributions of Heady & Co. just cited. For example: (a) The square-root functional form has never been put into harness in a mainstream microeconomics textbook, at least to this author's knowledge.[4] (b) In his literature review entitled Utility and Production Functions: Theory and Applications, Chung (1994) failed to mention the square-root functional form by name, and failed to cite any research by Heady & Co.

Remark 2–Our Special Case of Eq. (1): For the purposes of the present paper (purposes which we repeat are purely pedagogical), we advocate the imposition of three restrictions on Eq. (1), viz.,

$$\alpha = \delta_{ij} = 0 \quad \text{and} \quad \beta_i = 1 \tag{2}$$

for all i and j, plus the restriction that $n = 2$. Under such restrictions, the square-root production function becomes:

$$y = \sqrt{x_1} + \sqrt{x_2} \tag{3}$$

For added leverage, we next note that producer theory is widely recognized as the mirror image of consumer theory,[5] and as a consequence we next take the present opportunity to augment Eq. (3) with:

$$u = \sqrt{x_1} + \sqrt{x_2} \tag{4}$$

Eq. (4) denotes our square-root utility function, where u takes the place of y in Eq. (3) and where u denotes utility.

Remark 3–One Implication of Our Special Case: One might well ask, what does the assumption that $\delta_{ij} = 0$ [Eq. (2)] imply about our production function [Eq. (3)] and our utility function [Eq. (4)]? This restriction implies that the related functions

[3] The reader may wish to note that: (a) Earl O. Heady (1916-1987) was a professor of agricultural economics at the University of Iowa. (b) Heady produced 26 books and about 800 journal articles, research bulletins, and monographs. (c) During his career, Heady supervised 359 scholars from approximately 50 countries. (d) Heady is best known for his 1952 textbook, *Economics of Agricultural Production and Resource Use*. [Source: Wadsley and Heady Earl, 2009].

[4] It could be argued that there is one (minor) exception to this claim. In his *Introductory Mathematical Economics*, Hands (1991, pages 137–138) employed a special case of the square-root functional form [viz., our Eq. (3)] in his treatment of a two-factor-input production function. In doing so, we note that Hands did not offer an explicit justification for this functional form, nor offer a discussion of its historical or conceptual roots.

[5] This point was made by Cornes (1992, page 104) in his book, *Duality and Modern Economics*. He wrote: "From the point of view of duality theory, the ideas .. encountered in consumer theory carry over very naturally into the analysis of production. There are no fundamentally new ideas, simply old ones in a slightly different guise."