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Finding mixed strategy Nash equilibria with decision trees

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A B S T R A C T

This paper describes the usefulness of decision tree models for determining mixed strategy Nash equilibria in normal form games, particularly to undergraduate students. The approach is to construct a decision tree for each player, then solve the model via dynamic programming to determine the equations that must be satisfied at Nash equilibrium. This method not only provides a computational device that can be used to calculate the Nash equilibrium, but also serves as a visual aid that helps students understand the Nash equilibrium concept.

1. Introduction

As instructors, we search for alternative methods of explaining and solving problems that are difficult for our students to understand, and for ways to integrate their learning experience across related fields. This is our motivation for using decision trees to teach mixed strategy Nash equilibria.

With practice, most students comprehend techniques for finding pure strategy Nash equilibria in normal form games. However, students find it significantly more difficult to find mixed strategy Nash equilibrium solutions to normal form games. The decision tree method we describe in this paper provides instructors an option for explaining mixed strategies to students, and for training students to find mixed strategy Nash equilibrium (heretofore MSNE) solutions to normal form games.

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A typical undergraduate economics textbook presents normal form games with both pure strategy and mixed strategy equilibrium solutions (see, for example, Frank, 2010; Pindyck and Rubinfeld, 2012; Varian, 2009). These textbook solutions are typically purely algebraic (and not graphical) solutions and are often not intuitive to the student. Other approaches, such as that of taking the derivative of the player's (linear) expectation function (Baldani et al., 2005), seem even less intuitive than the typical algebraic solution. This experience seems not to be limited to our students alone (see, for example, Garrett and Moore, 2008). For this reason, we designed a decision technique to solve for a MSNE that allows students to visualise the solution.

The decision tree methodology we propose has the advantage that it does not require students to have a sophisticated mathematical background. It is intended for those students who are learning about a mixed strategy equilibrium for the first time. These are students who are in an upper-level undergraduate economics or business course (Microeconomic Theory, Managerial Economics, or Operations Management, for instance). Therefore, the only real mathematical tool they need, that of calculating expected values, should be well within their capability.

An additional benefit of this method is that it is an excellent way to integrate the structured approach to decision analysis, typically taught in operations management or management science courses, with the analysis of strategic behavior, introduced in economics courses.

In the next section we describe how to use decision trees to find a mixed strategy Nash equilibrium, and demonstrate the advantages of using this graphical technique. In Section 3, we present the use of the decision tree methodology for solving a game where some strategies are dominated. Finally, we conclude with a discussion of our classroom implementations of the decision tree method and mention potential applications in the decision sciences.

2. Mixed strategy equilibria in normal form games

It is our experience that although our students are able to comprehend and solve games with a pure strategy Nash equilibrium, the mechanics and intuition of finding a MSNE are harder to master. For this reason, we expose them to the decision tree technique as a way to solve for a MSNE that allows them to visualize the solution. By providing a useful visual tool to illustrate the mixed strategy Nash equilibrium concept to students, we believe that the decision tree approach has an advantage over existing methods for solving simple normal form games. Moreover, it can be extended to more complex games.

2.1. Normal form representation

We begin with a normal form representation of a two-player game $G=\{S_1, S_2; u_1, u_2\}$, defined by the ith player's strategy space S_i and payoff function u_i. A Nash equilibrium occurs where there is no incentive for a player to deviate from the chosen strategy. More formally, the strategies $(s_1{}^*, s_2{}^*)$, where $s_i \in S_i$, are a Nash equilibrium if $u_1(s_1{}^*, s_2{}^*) \geq u_1(s_1, s_2{}^*)$ for all $s_1 \in S_1$ and $u_2(s_1{}^*, s_2{}^*) \geq u_2(s_1{}^*, s_2)$ for all $s_2 \in S_2$.

In order to understand the difference in complexity in solving for a pure strategy versus a mixed strategy Nash equilibrium, consider the two-player, normal form game (Gintis, 2000) in Table 1, which represents the payoffs to each player for each possible strategy pair, with Player 1's payoff listed first. In this game, Player 1 chooses from its strategy set $S_1=\{L, M, R\}$, while Player 2 chooses from its strategy set $S_2=\{A, B, C\}$. In the example in Table 1, a strategy s_1 can be interpreted as an assignment of

Table 1
Example of a normal form game.

Player 1	Player 2		
	A	B	C
L	0, 0	4, −4	2, −2
M	1, −1	0, 0	2, −2
R	2, −2	1, −1	0, 0