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Early use of decomposition for addition and its relation to base-10 knowledge☆

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ABSTRACT

The early use of decomposition for addition has been linked to future mathematics achievement. The present study examined kindergartners' performance on addition problems, focusing on their use of the decomposition strategy and the factors related to the frequency with which they chose it. Single- and multi-digit addition problems were presented to kindergartners from US, Russia and Taiwan ($N = 182$). As expected, kindergartners used a variety of strategies to solve the problems. They were more likely to use decomposition on complex problems involving carryover or multi-digit operations. Critically, their use of base-10 decomposition was related to their knowledge of base-10 number structure. These relations were similar across all three nations. Implications for understanding mathematical development and designing early mathematics instruction are discussed.

A key goal of early mathematics instruction is to develop students' ability to efficiently and accurately perform basic arithmetic operations (National Council of Teachers of Mathematics [NCTM], 2000, 2006). Both researchers and educators have emphasized the importance of acquiring these skills in a meaningful way — in connection to a conceptual understanding of numbers and numeric relations, rather than mechanically learning a set of procedures. Thus, researchers have increasingly focused their investigation on the strategies children use when solving arithmetic problems and how children select among the different strategies available to them (Canobi, Reeve, & Pattison, 2003; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Geary, Bow-Thomas, Liu, & Siegler, 1996; Lindberg, Linkersdörfer, Lehmann, Hasselhorn, & Lonnemann, 2013).

Examining children's strategy choices provides insight into their understanding of the numeric structure and relations among numbers. For example, when children decompose the addends in an addition problem in order to simplify the computation, they demonstrate an understanding of the composition of numbers (e.g., $7 + 4 = 7 + 3 + 1 = 10 + 1 = 11$). Several studies have found a relation between the frequency with which children use more advanced and efficient mental computational strategies and mathematics performance. In particular, starting in first grade, use of a decomposition strategy has been associated with better performance on a variety of tasks that involve computation (Carr, Steiner, Kyser, & Biddlecomb, 2008; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Geary, Hoard, Nugent, & Bailey, 2013). For example, Fennema, Carpenter, Jacobs, Franke & Levi (1998) found that first graders who used the mental decomposition strategy demonstrated deeper conceptual understanding of addition and subtraction on transfer problems than those who did not rely on this strategy.

Thus, it is important to better understand early individual differences in the use of decomposition and the factors that contribute to those differences. In the present study, we examined the frequency with which kindergarten students used a base-10 decomposition strategy to solve addition problems. We were particularly interested in understanding whether and how these differences in strategy choice were related to problem characteristics and children's understanding of the base-10 numeric structure.

Addition strategies

Addition problems can be solved using a variety of strategies. Generally, children use three types of addition strategies: counting, decomposition, and retrieval (Geary, Bow-Thomas, Liu & Siegler, 1996; Geary, Fan, & Bow-Thomas, 1992; Shrager & Siegler, 1998). *Counting* is

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among the first to emerge in children's repertoire of addition strategies. To begin with, counting strategies involve enumerating both of the addends ("*count-all*"). This strategy has been observed in children as young as 3-year-olds. During the preschool years, children acquire a more advanced counting strategy, which involves counting up from one addend the value of the second addend ("*count-on*"; Siegler & Robinson, 1982). By the age of five, most children have mastered counting strategies. Through experience using these strategies, children begin to commit simple addition facts to memory, which leads to the emergence of memory-based strategies, such as retrieval and decomposition. The *retrieval* strategy involves recalling the solution to a problem as a number fact stored in memory, rather than active computation. *Decomposition* involves transforming the original problem into two or more simpler problems, using either a previously memorized number fact ("*fact-based decomposition*") or the base-10 properties of the number system ("*base-10 decomposition*"). An example of the latter is solving $6 + 5$ by adding 6 and 4 to get to 10, and then 1 more; an example of the former is using a memorized fact $6 + 6 = 12$, and then subtracting 1 to solve $6 + 5$. Children's skill in executing these strategies continues to improve over the elementary school years, gradually being extended to more complex problems.

Children are generally able to use more than one strategy to solve arithmetic problems (Ashcraft, 1982; Carpenter & Moser, 1984; Geary, 1994; Siegler & Shrager, 1984). For example, when asked to solve a series of problems in one session, a child might count-on to solve one problem, retrieve the answer from memory to answer the next problem, and use decomposition to solve another problem. Yet, among children of the same age there are individual differences in terms of the frequency with which they use different strategies. Further, with development, the extent to which children rely on particular strategies changes such that the frequency of counting strategies decreases and the use of retrieval and decomposition increases (e.g., Carpenter & Moser, 1984; Fuson, 1992; Geary, Hoard, Byrd-Craven & DeSoto, 2004; Siegler & Shrager, 1984).

Strategy choice

Children's adaptive selection of strategies is a key process underlying development in mathematics as well as other domains (Siegler, 1996). The Overlapping Waves Theory of development posits that (a) at any one time, children know and are able to use multiple strategies to solve problems in a given domain; (b) these strategies compete with each other over prolonged periods of time; and (c) cognitive development and improved performance in a domain involves changes in the relative frequency of use of these strategies, with new strategies sometimes being added and others sometimes ceasing to be used.

This theory describes strategy choice as a competition between accuracy and efficiency (Shrager & Siegler, 1998). Early on children's predominant strategy may be a relatively inefficient one, such as count-all, that leads to a correct response. As children learn new strategies and are able to execute them correctly, their predominant approaches change toward more efficient strategies. At any given time, strategy choice is constrained by characteristics of the problem being solved, such as problem difficulty, and of the individual solving the problem, such as prior knowledge (Geary, Hoard, Byrd-Craven & DeSoto, 2004; Kerkman & Siegler, 1993; Laski et al., 2013; Lemaire & Callies, 2009; Siegler, 1988).

Problem characteristics

The frequency with which children use a particular strategy is related to problem difficulty. Children tend to use retrieval on easier problems for which it is likely to lead to an accurate response, but select an alternative strategy that involves computation on more difficult problems for which retrieval is less likely to lead to an accurate response (Lemaire & Callies, 2009; Siegler, 1996).

The likelihood of choosing a specific computational strategy, thus, varies with problem characteristics. Evidence suggests that children are unlikely to use the most advanced computational strategy available to them unless the difficulty of the problem demands it. Increasing problem difficulty promotes the use of more advanced computational strategies, in order to maximize efficiency while still maintaining accuracy. For instance, Siegler & Jenkins (1989) investigated the use of counting strategies in preschoolers. They found that the frequency with which children used the more advanced count-on strategy (as opposed to count-all) was higher when children were presented with complex problems involving an addend above 20 than when they were presented with simple problems involving addends between 1 and 5.

Given the findings about what influences preschoolers' choice of counting strategies, it is possible that older children, having a better understanding of numeric structure, would be more likely to use a decomposition strategy on problems for which counting is relatively inefficient (e.g., $26 + 8$). In fact, a cross-national study by Geary, Bow-Thomas, Liu & Siegler (1996) showed that Chinese kindergartners used decomposition more frequently on problems with sums greater than 10 than on problems with sums less than 11. At the same time, their American peers did not show a similar pattern as a function of problem difficulty. It should be noted, however, that children in this study were given addition problems involving only single-digit addends. In the present investigation, we extended the range of problems to include mixed-digit addition, where one of the addends is a single-digit and the other is a double-digit number, to explore whether these problems push kindergartners to use a decomposition strategy more often than in previous studies.

Students' prior knowledge

While problem difficulty can affect the frequency with which children use different strategies, children's strategy choice is also constrained by individual differences in knowledge that may be necessary to execute certain strategies (Geary, Hoard, Byrd-Craven & DeSoto, 2004; Imbo & Vandierendonck, 2007; Kerkman & Siegler, 1993; Torbeyns, Verschaffel, & Ghesquiere, 2004). For example, in the Siegler & Jenkins (1989) study, increased problem difficulty promoted preschoolers' use of the count-on strategy only among those children who had already demonstrated an ability to execute the strategy on easier problems. Similarly, the differences in the use of decomposition between Chinese and American kindergartners observed in prior work (Geary, Bow-Thomas, Liu & Siegler, 1996) could reflect differences in prior mathematical knowledge and experience. We propose that one aspect of numerical understanding that may contribute to children's use of base-10 decomposition strategies, which are the focus of the present paper, is their understanding of the base-10 structure of the number system.

Base-10 knowledge

There is general agreement that knowledge of the base-10 system is a critical aspect of mathematics performance (Geary, 2006; Miura, 1987; National Research Council, 2001; NCTM, 2000). More specifically, it is widely believed that base-10 knowledge is related to the accuracy of children's computation of multidigit arithmetic problems (Fuson, 1990; Fuson & Briars, 1990; National Research Council, 2001). Errors in carrying and borrowing in written addition problems, for instance, have been attributed to a lack of understanding of base-10 and place value (Brown & Burton, 1978; Fuson, 1990; Hiebert, 1997; Ross, 1986; Valeras & Becker, 1997).

Children learn the structure of the base-10 system and the place-value notation for whole numbers gradually over the course of several years. Even before formal instruction, children are exposed to key aspects of base-10 knowledge. For example, they are exposed to counting beyond 10 and the repeating decade structure of number words. Indeed, the length of children's count string increases between