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Disappearing x: When solving does not mean finding the solution set

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ABSTRACT

This study deals with preservice elementary teachers' responses to linear equations and inequalities that had infinite solution sets. In particular, these tasks dealt with situations where the variable is eliminated during standard symbolic manipulation. The results reveal that infinite solution sets proved difficult for the participants, particularly when prompted to solve the linear equations and inequalities. When the direction prompt was changed, there was increased success in finding the correct solution set. The directional prompts changed the types of solution strategies as well as the nature of responses. In addition, participants stated the belief that different, mathematically equivalent prompts required different, non-equivalent, types of solutions and allowed for different solution strategies.

1. Introduction

There has been a growing movement to view algebra as a necessary component of kindergarten through 12th grade education. The National Council of Teachers of Mathematics listed algebra as one of its main standards for K-12 mathematics education (NCTM, 2000). Currently, simple missing addend problems are being introduced in the first grade with the implementation of the Common Core State Standards (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). In addition, the RAND Mathematics Study Panel identified the teaching and learning of algebra in grades K-12, along with teachers' mathematical knowledge, as two of the most pressing issues in mathematical proficiency (Ball, 2003).

How preservice teachers are introduced to these introductory linear equations can have a profound impact on how they view them in the future. "What prospective teachers bring to teacher education programs is a critical influence on what they actually learn there" (Ball, 1988, p. 15). If these conceptions are not understood and addressed, then preservice teachers are unlikely to move beyond them and be able to effectively teach (Thanheiser, 2009). If, for example, they have a very limited, procedural, view of what solving an equation means, this will impact their future students.

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2. Review of the literature

Studies have shown that preservice elementary teachers view algebra as a subject that mainly deals with symbols and symbolic manipulation. Sample students were judged to have “done algebra” if their work involved symbolic manipulation rather than the thinking involved in how the students solve the problems (Stephens, 2008).

Equations, and specifically the equal sign, have received a great deal of attention by researchers (Asquith, Stephens, Knuth, & Alibali, 2007; Hallagan, 2006; Hattikudur & Alibali, 2010; E. Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005; E.J. Knuth, Stephens, McNeil, & Alibali, 2006; Stephens, 2006, 2008). Research has shown that a common misconception is to view the equal sign as an input-output signal, as in three plus five produces (equals) eight (Kieran, 1981). In other words, many students have an operational view of the equal sign. The equal sign may be is viewed as “the total” or “the answer” (McNeil & Alibali, 2005). This operational view can lead to mistakes such as answering 17 when trying to solve the number sentence: $8+4=\blacksquare+5$ (E.J. Knuth et al., 2006). In contrast, a relational understanding of the equal sign recognizes that the equal sign is a symbol for mathematical equivalence; that both sides must balance (A.C. Stephens, 2006). Instruction can affect the type of thinking, operational vs. relational, a student has. Preservice mathematics textbooks frequently have little explanation on how to teach students about the equal sign. Even worse, some recommendations may encourage operational thinking instead of relational thinking (Hattikudur & Alibali, 2010).

Although inequalities are frequently grouped together with equations, this can be problematic and for the purpose of this study. It is important to recognize the distinction between the two. Hattikudur and Alibali (2010) found that learning about inequalities could encourage relational thinking regarding the equal sign. However, the converse is not necessarily true. Students frequently treat inequalities as if they were equations and are often encouraged to do so, with varying degrees of success. The balance model, which is a useful and powerful tool for equations, is not as successful for inequalities (Vaiyavutjamai & Clements, 2006). After all, inequalities are inherently unbalanced.

A standard procedure for dealing with a linear equation or inequality involves performing like operations on both sides of the equation until a variable is isolated on one side. This procedure may frequently obtain the correct answers, for both equations and inequalities. It is unclear, though, whether students are aware that sometimes the solution set was an infinite set or even what their final statement meant. Previous studies have found that students frequently do not know how to check their answers with an inequality and their typical solution strategy is characterized by using rules without reason (Vaiyavutjamai & Clements, 2006).

However, very little research has been done on inequalities and more is needed (Almog & Ilany, 2012; Tsamir & Almog, 2001; Tsamir & Bazzini, 2004; Vaiyavutjamai & Clements, 2006; Warren, 2004). The limited research available has shown that so-called strange solution sets to inequalities, such as all real numbers, the empty set, and a single value solution can be especially problematic for students (Almog & Ilany, 2012; Fujii, 2003; Tsamir & Bazzini, 2004). Tsamir and Bazzini (2004) found that only half of their participants were able to correctly claim that a single value could be the solution set for an inequality. Significantly fewer participants were able to come up with a correct inequality that would have a single value as a solution set. Two intuitive beliefs could be contributing to these difficulties: the belief that inequalities must result in inequalities and that solving inequalities is the same process as solving equalities (Tsamir & Bazzini, 2004). Marjanovic and Zeljic (2006) advocate spending a larger amount of time on inequalities in the early grades since inequalities have solutions that typically have more than a single number. In fact, the solution sets are frequently infinite.

Substantial attention has been paid to the effects that comparing solution strategies have on students (Lampert, 1990; Rittle-Johnson & Star, 2007; Sierpinska, Bobos, & Pruncut, 2011; Yakes & Star, 2011). For example, there is some evidence that presenting multiple solution strategies to students can increase their “flexibility of thought” in solving absolute value equations (Sierpinska et al., 2011, p. 275). Procedural flexibility involves knowledge of multiple solution strategies and when to use each of them (Rittle-Johnson & Star, 2007; Star, 2005). Comparing solution methods has been shown to make students’ and in-service teachers’ procedural flexibility increase (Rittle-Johnson & Star, 2007; Yakes & Star, 2011).

3. Purpose of the study

This study examined how preservice elementary teachers responded when the solution set was infinite for an equation, a somewhat unusual situation for a linear equation, and when the same solution set was true for a linear inequality. In both cases, when typical symbolic manipulation is used, the variable cancels, or as some participants observed, it “disappears.” Some of the participants were then selected for interviews where the same tasks were given to them again, but this time the prompt was changed in an attempt to draw more attention to the solution set and allow for the possibility of multiple solutions.

4. Data collection and analysis

The tasks discussed here were both administered as part of a larger study designed to test participant’s basic skill level with literal symbols in different usages and contexts as well as elicit short written statements that were analyzed for common themes. Data were collected through two different means. First, a written questionnaire was administered to 58 preservice elementary teachers at a medium sized, public, western university. These written questionnaires were given to students attending any section of the mathematics content courses designed for preservice elementary teachers during the