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Temporal freedom in mathematical thought: A philosophical–empirical enquiry

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ABSTRACT

This paper seeks to introduce and elaborate on the notion of '*temporal freedom*' in the context of mathematical thinking.

The intention is to merge a philosophical and empirical discussion on this topic, and findings will be presented from a qualitative study in which 29 high-school students were invited to explore the symmetry group of a square during clinical interviews. The data indicate how the interviewees – who were given the opportunity to freely explore an unseen mathematical system – repeatedly brought past and future together in a tangible present during their investigations. In doing so they were exceptionally successful in engaging with advanced abstract algebra, far beyond what might have been expected at the outset.

By using empirical data to exemplify an otherwise highly philosophical discussion, I hope to draw explicit attention to the temporal aspect of the human mind in mathematical action which instructional methods frequently fail to recognise, value and utilise.

1. Introduction

The aim of this paper is to introduce and elaborate on the notion of '*temporal freedom*' in the context of mathematical thinking.

The idea to address this issue emerged during a study in which 29 high-school students (aged 16–21) were invited to explore the symmetry group of a square (some individually and some in pairs) during video-recorded clinical interviews. These students had no previous knowledge of group theory, and thus the recorded data allowed for a detailed examination of their developing understanding of a mathematical system, from its very foundations (based on manipulations of a physical square), to some of its highly intricate algebraic characteristics.

The interviews lasted up to two hours, but as the data were being analysed, it became increasingly clear that 'two hours' was a wholly inadequate way of addressing the aspect of *time* involved in these mathematical enquiries. The ways in which the interviewees approached the task, the ways in which their early ideas were used and revised, the ways in which their hypotheses were projected into the future, caught up with and adjusted at a later stage, and not least the ways in which they paused in the face of a mathematical revelation, led me to believe that there was a temporal aspect to their explorations that lived a life unchained from linear clock-time measured in minutes from zero to 120.

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The task was certainly not designed with the notion of temporality in mind, but it was developed for and largely *by* the students, and in the freedom that they were given in exploring and expanding the task, in the interest that was shown in their thinking, and in the opportunity to engage with the same task for an extended period of clock-time with no demands on linear progression of facts, the students' temporality emerged and stood out in its own right. I would go as far as to attribute the exceptional mathematical success that was had in dealing with the task, by vocational and academic track students alike, to this factor of 'temporal freedom'.

However in seeking to describe and capture this aspect of their work with reference to existing literature, there seemed to be a distinct lack of explicit discussions of time and temporality in the context of mathematical thought. By this I certainly do not mean to say that the concept of time is wholly lacking from educational theories, but rather that when it comes to the individual act of doing mathematics – on a micro-level so to speak – the issue is in need of further attention.

It could for example be argued that sociocultural and situative perspectives on mathematics education pay attention to time by highlighting that the work of mathematicians is always part of a particular cultural practice, social and historical in nature, and in this sense, not atemporal. Lave and Wenger (1991) have argued that learning is a social process whereby knowledge is co-constructed, situated in a specific context and embedded within a particular social and physical environment. All of these factors undeniably emerge *through*, *in* and *of* time, and if one accepts that mathematical activity takes place in cultural contexts, is mediated by language and other symbol systems, and is best understood with an eye to its historical development, then the importance of time is almost embraced by default. However this takes a somewhat macro-level perspective on the matter, and does not speak (at least not to my understanding) to the temporality of the individual act of doing mathematics, on the micro- or even phenomenological level.

Constructivist perspectives do perhaps come closer to the matter by specifically addressing how previous knowledge is constantly retrieved and reconstructed when new problems are faced and new knowledge required, thereby pointing to the non-linear and iterative ways is which mathematical understanding is developed. Yet I have struggled to find any literature that unwraps this process with explicit reference to time and human temporality.

In the following I will discuss what is meant here by temporality – as a concept that is distinct from so called 'clock-time', and I will then proceed to present specific examples from empirical data which in turn will be used to shed light on the notion (and importance) of temporal *freedom*.

2. Temporality as distinct from 'clock-time'

In classical physics we find a notion of time as an arrow of infinitesimal moments which flow in a constant stream. In other words, time is conceived of as a uniform and linear series of 'now-points', a conception in which the future is the 'not-yet-now', the past is 'no-longer-now' and the present is always flowing from future to past. In some sense this corresponds to our understanding of 'clock-time', which is constantly moving in one direction – from the past, through a mercilessly uncapturable, always escaping present, and to the future.

Now whereas this model of time can prove useful in classical physics, applying it in enquiries into the functionings of the human mind and experience, can prove highly problematic. Many reasons could be put forward for this, but there are two that most readily come to mind. Firstly, this conception of time effectively reduces the present – where we can act as free agents – to something infinitely small, on the border of non-existence. Secondly, it imposes a kind of determinism where the past becomes an *irrevocable* determinant of our (barely existing) present and of our future. One might certainly recognise this kind of determinism from classical physics where cause precedes effect, linearly and unquestionably. Thus our 'clock-time present' is effectively placed on an imaginary line, squeezed from one side by the past into and onto a determined and non-budging future, rendering our 'now', where we live, act and *exist* so small and under so much pressure that it becomes more of a theoretical idea than a tangible phenomenon.

This view is understandably unfortunate in the context of education in general and the learning of mathematics in particular. A time in which the past is an irrevocable determinant of the future and where no graspable present can be found is bad news for many if not most students of mathematics. For those who may struggle with the subject it becomes downright hopeless, because an irrevocable past does not allow for a present or indeed a prospect of a future with genuine opportunities to improve.

However a number of objections have been made to this linear view of time (which Heidegger, 1962 notably referred to as 'vulgar time') in the context of human experience, and a defence of the present as more than 'almost nothing' has been put forward in various ways. For instance the 'specious present', a term coined by James (1890), and later described by Husserl (1999, p. 42) as "the rough now" has been used to denote a view of the present that differs significantly from the infinitesimal 'now-points' of clock-time. For James contended that

> *the practically cognized present is no knife-edge, but a saddle-back, with a certain breadth of its own on which we sit perched, and from which we look in two directions into time. (1890, p. 609)*

Or as Franck (1994) has phrased it,