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ICT-supported problem solving and collaborative creative reasoning: Exploring linear functions using dynamic mathematics software

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ABSTRACT

The present study investigates how a dynamic software program, GeoGebra, may support students' collaboration and creative reasoning during mathematical problem solving. Thirty-six students between the ages of 16 and 17 worked in pairs to solve a linear function using GeoGebra. Data in the form of recorded conversations, and computer activities were analyzed using Lithner's (2008) framework of imitative and creative reasoning in conjunction with the collaborative model of joint problem space (Roschelle & Teasley, 1994). The results indicated that GeoGebra supported collaboration and creative reasoning by providing students with a shared working space and feedback that became the subject for students' creative reasoning. Furthermore, the students' collaborative activities aimed toward sharing their reasoning with one another enhanced their creative reasoning. There were also examples of students using GeoGebra for trial-and-error strategies and students engaging in superficial argumentation.

1. Introduction

One of the challenges in mathematics education is helping students to become skilled problem solvers, rather than rote learners. A research framework presented by Lithner (2008) describes how rote learning relates to students' line of thinking or reasoning. Reasoning based on rote learning is categorized as *imitative*; during lectures, students memorize facts and algorithms and subsequently attempt to recall them when solving tasks. Conversely, *creative reasoning* engages students in instructive problem-solving processes, during which they develop well-founded and mathematically anchored arguments for their choice of methods. Studies have shown that students who engaged in creative mathematical reasoning to solve non-routine problems during a training session performed significantly better on post-tests than students who used imitative reasoning when working with repetitive tasks (Jonsson, Norqvist, Liljekvist, & Lithner, 2014). Other similar studies have shown, based on post-test results, that students who work with complex problems outperform students who are given traditional lectures and practice well-structured tasks (Boaler, 1998; Kapur, 2011).

Problem solving related to functions (e.g., linear or polynomial) is no exception to this trend; students tend to use imitative reasoning and superficial argumentation when they find this type of mathematics difficult (Even, 1998; Hoffkamp,

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2011). Similar findings as above have been reported for this type of problem solving. Non-procedural assignments provide students with opportunities to challenge their understanding of relations instead of performing procedures (Ferrara, Pratt, Robutti, Gutierrez, & Boero, 2006, Mevarech & Stern, 1997). Moreover, several studies emphasize the value of collaborative work. Students' verbalization of mathematical concepts to engage in dialog has been shown to be beneficial to enriching their conceptualizations (Hoffkamp, 2011) or establishing mathematical meaning (White, Wallace, & Lai, 2012). However, there are studies that problematize these findings, pointing out obstacles to working with non-routine problems without supporting activities (Ploetzner, Lippitsch, Galmbacher, Heuer, & Scherrer, 2009) and issues with students working in groups. The latter issue refers to students' tendency to cooperate, dividing work amongst themselves, rather than collaborate, sharing understanding and solving the problem together (Roschelle & Teasley, 1994).

Research provides various methods for supporting students in developing conceptual understandings as well as collaborative work. One of the suggested methods is the use of dynamic software that allows students to visualize functions and their representations (Rakes, Valentine, McGatha, & Ronau, 2010), as well as distribute their collaborative problem solving process (Stahl, Koschmann, & Suthers, 2006).

The idea of considering the appropriate support for student engagement in collaborative problem solving and creative reasoning combined with the proposition that technology may support these activities bring us to the following question: How can dynamic software (in this case, GeoGebra) support or obstruct students' creative reasoning and collaborative work during the problem-solving process?

1.1. Aim and research questions

The aim of this study is to develop insight into how GeoGebra could be used as a means of supporting collaboration and creative reasoning during a problem-solving process.

The following research questions will be addressed in this study:

To what extent do students use GeoGebra to collaborate during problem solving?
What characteristics of GeoGebra might contribute to or obstruct their creative reasoning?

To examine how GeoGebra may support students' collaboration and creative reasoning, the didactical situation in this study was designed to allow students to work in pairs to solve non-routine tasks while supported by GeoGebra. The didactical situation was designed to be in line with Brousseau and Schoenfeld's suggestions, which will be presented in the following section along with the theoretical frameworks used to analyze data.

2. Research framework

The following section begins by introducing the theoretical concepts used to design the didactic situation in this study, followed by a presentation of the theoretical frameworks for creative reasoning (Lithner, 2008) and collaboration (Roschelle & Teasley, 1994). The latter will be used for structuring and analyzing the data.

2.1. Designing a didactic situation, creative reasoning and collaboration

Students spend much of their time completing textbook exercises in which examples are followed by similar tasks. Thus, students are guided into imitative reasoning that does not give them an opportunity to argue for their strategies (Boesen, Lithner, & Palm, 2010). Solving tasks using imitative reasoning may result in correct answers; however, to develop a conceptual understanding, students need to process mathematical concepts—to struggle, in a productive sense (Hiebert & Grouws, 2007). Schoenfeld (1985) argues that learners need to work with mathematics problems that are somewhat new to them. When students engage in challenging problem solving, they need, and therefore develop, their mathematical knowledge and understanding as well as their ability to create strategies for working on unfamiliar problems. That is, to engage in creative reasoning, students need to work with non-routine tasks for which they have no memorized procedure to imitate to solve the task (Lithner, 2008).

Furthermore, to be engaged in creative reasoning, students need to struggle with the problem without guidance toward a correct solution. In line with the idea that imitating procedures is inefficient for learning, Brousseau (1997) suggests a didactical design that leaves some of the responsibility of the problem-solving process to the students. During this part of the didactical situation, defined as an adidactical situation, teachers should not interfere or guide students toward the desired answer. However, the adidactical situation should involve feedback related to the students' actions. Brousseau (1997) refers to feedback as "*an influence of the situation on the pupil*," suggesting that the situation will provide each student with influence "*as positive or negative sanctions relative to her action, which allows her to adjust this action, to accept or reject a hypothesis*" (Brousseau, 1997, p. 7).

In conjunction with challenging problems and adidactical situations, collaboration is often suggested as an alternative to traditional methods (Boaler & Greeno, 2000; Stahl, Rosé, & Goggins, 2011). Students may improve their conceptual understanding from collaboration by engaging in discussions, mutual explanations, and elaboration of underlying mathematical concepts (Mullins, Rummel, & Spada, 2011, Scardamalia & Bereiter, 1994). However, having students work in small groups