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Discovering and addressing errors during mathematics problem-solving—A productive struggle?

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ABSTRACT

The present study investigates students' struggles when encountering errors in problem-solving. The focus is students' problem-solving activities that lead to productive struggle and what the students might gain therefrom. Twenty-four students between the ages of 16 and 17 worked in pairs to solve a linear function problem using GeoGebra, a dynamic software application. Data in the form of recorded conversations, computer activities and post-interviews were analyzed using Hiebert and Grouws' (2007. *Second handbook of research on mathematics teaching and learning* (Vol. 1). 404) concept of productive struggles and Schoenfeld's (1985. *Mathematical problem solving*: ERIC) framework for problem-solving. The study showed that all students made errors concerning incorrect prior knowledge and erroneously constructed new knowledge. All participants engaged in superficial, unproductive struggles moving between a couple of Schoenfeld's episodes. However, a majority of the students managed to transform their efforts into productive struggle. They engaged in several of Schoenfeld's episodes and succeeded in reconstructing useful prior knowledge and constructing correct new knowledge—i.e., solving the problem.

1. Introduction

To learn mathematics and develop problem-solving skills, students must work with mathematical problems, i.e., novel tasks and intellectual challenges that cannot be solved by merely imitating memorized procedures (Schoenfeld, 1985). That is, instead of following a given procedure on how to solve a problem, students must engage in productive struggle and create at least parts of their own methods (Hiebert & Grouws, 2007). However, because procedures are designed to circumvent meaning and provide students with fast and accurate ways to solve a problem (Brousseau, 1997), it is likely that that students who create their own problem-solving methods make more errors and engage in more time-consuming struggles to address those problems than students using memorized procedures. These circumstances may account for some explanation why instruction in problem-solving procedures appears to dominate classroom activities (Hiebert & Stigler, 2004). Teachers instruct with the objective of preventing students from spending time engaged in activities that would appear as unproductive (Smith, 2000). That is, teachers help students avoid time-consuming struggles, for example, when they are addressing their errors, by providing them with accurate methods to complete the tasks and thereby eliminating "problem-solving" (Santagata, 2005; Stigler & Hiebert, 2009).

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However, studies have shown that students who construct their own methods, even though their attempts may include errors, perform better on tests than students using instructed procedures. The former student group outperforms the latter on posttests (Jonsson, Norqvist, Liljekvist, & Lithner, 2014). Even students who create their own methods but fail to solve the problem, i.e., reaching a specific learning target during training, score higher on posttests than students who use procedures taught in the classroom (Kapur, 2011, 2014). Furthermore, students who fail are more likely to evaluate their methods than students who succeed (Granberg & Olsson, 2015). It appears that making, discovering and correcting errors may generate effort that can engage students in productive struggle. However, if a teacher considers situations in which students are making and addressing errors as merely unproductive, it is likely that the teacher will provide the students with instructions on how to solve the task, rather than support students in becoming productive strugglers. It is a true challenge for teachers to distinguish productive struggle from unproductive and knowing when and how to step in to help students (Verkaik & Ritsema, 2006). It is furthermore not fully established how to translate the abstract idea of *struggles* into specific artifacts and activities. Therefore, it would be helpful to better understand how to identify the activities students engage in during productive and unproductive struggles. Furthermore, it would be valuable to obtain additional insight into what students might gain from their struggle when they address their errors, in other words, to what extent their struggles with errors could be described as beneficial, i.e., as productive struggles.

This study will look closely at students' activities and potential gains therefrom when they are struggling with their errors. The aim and research questions are presented in greater detail after the research framework is presented.

2. Research framework

The theoretical concepts that will be used to analyze the data are presented in the following section, starting with Hiebert and Grouws' (2007) concepts of struggle and productive struggle during problem-solving. Thereafter, Schoenfeld (1985) framework of problem-solving activities are presented.

2.1. Struggle

Hiebert and Grouws (2007) identified two features emerging from the literature as critical to developing conceptual understanding of mathematics: learning about mathematical concepts during lectures and struggling with these key concepts. In this sense, struggle indicates that students are engaged in deciphering mathematical concepts—i.e., attempting to understand concepts that are not immediately obvious to them. "If understanding is defined as the mental connections between mathematical facts, ideas and procedures, then struggling is viewed as a process that reconfigures these things" (Hiebert & Grouws, 2007, p. 388). The struggle is initiated when students' prior knowledge is insufficient to understand or address the given problem or the students are unable to assimilate new information. If their present comprehension is insufficient, they may need to reexamine and, if necessary, restructure what is already known. When a problem contains an unfamiliar element, students construct interpretations of the new information to connect to their present understanding and, if necessary, reform their prior knowledge or reinterpret what is new (Hiebert & Grouws, 2007). Hence, the struggle includes handling insufficient prior knowledge (e.g., correcting or reconstructing prior knowledge), as well as creating an interpretation of new information and constructing new knowledge in relation to what is already known.

A successful, productive struggle would result in the restructuring of mental connections in more powerful, useful ways through which the problem at hand would make sense and new information, ideas and facts would become assimilated (Hiebert & Grouws, 2007). Therefore, in this study, all activities that provide students more useful insights needed to solve (parts of) a given problem are defined as productive.

A number of researchers have addressed the question regarding how to conceptualize and describe the activities involved in the problem-solving process. Some of this research is based on the work of Polya (1945), who introduced four problem-solving principles: understand the problem, plan, act and check. Schoenfeld (1985) has developed this concept further and suggested five problem-solving episodes, which are presented in the following section.

2.2. Problem-solving activities

Schoenfeld (1985) identified different behavioral phases, i.e., activities, that take place during mathematical problem-solving, such as *reading* the given problem, *analyzing* the problem, *exploring* (e.g., choosing appropriate prior knowledge), *planning* how to solve the problem, *implementing* the plan and finally *verifying* the answer. The time spent on, for example, *reading* is defined as an *episode* of reading. Thus, problem-solving can be depicted as a process that starts with an episode of reading the given problem and ends with a solved problem if successful (Fig. 1). Furthermore, Schoenfeld observed various ways of engaging in the problem-solving process, which depend on whether the problem solver is a novice or an expert. Experts move back and forth between episodes, e.g., after verifying that a proposed solution is erroneous, they tend to revisit the episodes of reading, analyzing and exploring to develop alternative solving methods. Novices tend to omit certain episodes and often focus on fruitless ideas for a protracted period of time.

In this study, the *exploring* episode, including activities such as choosing and activating appropriate knowledge, is further specified regarding *prior knowledge* and *new knowledge* (Fig. 1) to include Hiebert's concept of struggle concerning activities of