Journal of Mathematical Behavior 42 (2016) 49–68

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Preservice teachers' response and feedback type to correct and incorrect student-invented strategies for subtracting whole numbers

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ARTICLE INFO

Article history:
Received 14 October 2014
Received in revised form 7 January 2016
Accepted 23 February 2016
Available online 7 March 2016

Keywords:
Whole number subtraction
Student strategies
Content knowledge
Pedagogical content knowledge
Preservice teachers

ABSTRACT

Despite the important role of teachers' interpreting and responding to student ideas in teaching mathematics, there are many unanswered questions related to teacher learning and the role of errors in student-invented strategies. This study examined the reasoning and responses of 140 preservice teachers (PSTs) to students' correct and incorrect strategies for whole number subtraction, as well as the PSTs' perceived challenges in connecting student strategies to traditional methods. The study also investigated the role that PSTs' specialized content knowledge plays in their ability to respond to correct and incorrect student-invented strategies, with respect to student-centeredness and feedback type. Results reveal that although some PSTs interpreted the validity and generalizability of the student methods incorrectly and justified them from the procedural aspect of each method, PSTs' specialized content knowledge (e.g., the mathematical depth of the PSTs' interpretation of student work) seemed to relate to their tendency to create student-centered approaches. However, specialized content knowledge did not seem to support PSTs' feedback type in the domain of whole-number subtraction.

1. Introduction

Interpreting and responding to student thinking is highlighted as one of the central tasks of effective mathematics teaching (National Council of Teachers of Mathematics [NCTM], 2000). Carpenter, Fennema, and Franke (1992) asserted that students who invent strategies or adopt them from classmates are intimately involved in the process of making sense of mathematics, and thus gain confidence in their abilities. Furthermore, the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) stress that teachers need to spend a significant amount of time and effort on student-invented methods or the informal strategies that arise in a typical mathematics classroom, and think about how to help students build on those methods before they introduce standard algorithms. However, it is not always easy for teachers to understand students' mathematical ideas, especially when such ideas differ from conventional mathematics.

In this study, I set out to investigate preservice teachers' (PSTs) reasoning, their responses to correct and incorrect student-invented strategies involving whole number subtraction, and the relationship between their knowledge and approaches. Although a growing body of research has explored teachers' knowledge and their teaching approaches in whole number

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http://dx.doi.org/10.1016/j.jmathb.2016.02.003

US traditional method	Common incorrect strategy	Partial differences strategy
62 - 25 5 12 62 -25 37	62 - 25 40	62 - 25 - 3 40 37

Fig. 1. An example of student-generated strategies in a subtraction problem.

computation (e.g., Empson & Junk, 2004; Flowers, Kline, & Rubenstein, 2003; Hill, Ball, & Schilling, 2008; McClain, 2003; Thanheiser, 2009; Thanheiser, 2010), there are many unanswered questions in relation to the role of teacher knowledge, teacher learning, and PSTs' interpretations of and responses to student-invented strategies.

First, the relationship between teacher knowledge and teachers' ability to recognize and respond to children's mathematical thinking and understanding is inconclusive, and at times even contradictory. While some studies reported a strong intersection between teachers' content knowledge (CK) and their ability to evaluate and respond to children's mathematical work (Baumert et al., 2010), others showed that strong CK does not necessarily lead to the development of PSTs' ability to analyze students' strategies (Son & Sinclaire, 2010; Bartell, Webel, Bowen, & Dyson, 2013; Peterson & Leatham, 2010). For example, Bartell et al. (2013), in examining the role CK plays in PSTs' ability to recognize children's responses (i.e., specialized content knowledge), reported that CK is not sufficient for supporting PSTs' analyses of children's thinking. As Rowland and Ruthven (2011) suggested, more research needs to be done on what sort of mathematical knowledge informs more effective teaching.

In addition, although a growing body of research has focused on teachers' treatment of student work, including student errors (e.g., Son & Sinclaire, 2010; Schleppenbach, Flevares, Sims, & Perry, 2007), PSTs' interpretation of and response to student-generated strategies, particularly incorrect student-generated strategies, has received limited attention in the research literature. Prior research has documented that teachers tend to perceive student errors as "dead ends" and thereby avoid and hide them in class (Borasi, 1994; Santagata, 2005; Stevenson and Stigler, 1992; Schleppenbach et al., 2007). Son and Sinclaire (2010), for instance, who explored PSTs' responses to student errors in the domain of geometry, reported that PSTs tended to resort to telling and showing (e.g., teacher-centered approaches) while focusing on procedural aspects of the mathematics, even though they recognized the source of student errors from its conceptual aspect.

The NCTM (2000) stresses that teachers should move beyond a superficial "right or wrong" analysis of student work, and use student errors as potential avenues for student learning. In light of the importance of student-invented strategies, it is important to ask whether PSTs' responses to incorrect student strategies are similar to their responses to correct student strategies. In particular, given the benefits of student-centered approaches over teacher-centered approaches (e.g., Boaler & Staples, 2008), of positive statements (praise) over criticism, and of effort feedback (i.e., praise for process or effort) over ability feedback (i.e., praise for intelligence) (e.g., Dweck, 1999; Kamins & Dweck, 1999), it is necessary to evaluate the approaches and feedback PSTs intend to use in responding to correct and incorrect student-generated strategies.

If teachers are called to use student thinking, including student errors, as springboards for inquiry into mathematical concepts, exploring these responses can help teacher education programs prepare PSTs to make better use of student-generated strategies. However, few studies have comparatively analyzed correct and incorrect strategies' impact on PSTs' interpretation of and responses to student thinking, and more specifically the role PSTs' specialized content knowledge plays in their ability to respond to correct and incorrect student-invented strategies with respect to student-centeredness and feedback type (Son, 2013).

Furthermore, proportionally limited attention is given in the research literature to supporting PSTs' teaching in connecting student strategies to a standard algorithm for whole-number subtraction, particularly in terms of the challenges PSTs might face when doing so. Researchers have documented that students tend to create their own strategies, different from traditional methods, when solving a regrouping subtraction problem (Carpenter et al., 1992; Carroll, 2000; Fuson, 1988; Selter, 2001). For example, in the problem 62−25, rather than using the standard algorithm of regrouping the 6 tens and 2 ones as 5 tens and 12 ones, some students decompose the numbers based on the place value, subtract each part separately without regrouping, and add the partial differences, as shown in Fig. 1.

Although one might think that knowing student strategies is enough, teachers also need to know and be able to explain conceptual relationships. Given that the traditional algorithm is known to be efficient and effective, teachers ultimately need to help students use that algorithm for later grades and explore its connection to student-invented strategies. However, we know little about how PSTs attempt to connect student strategies to standard algorithms, what type of feedback they plan to use in responding to student-generated strategies, what challenges they encounter, and what factors influence their responses to students' non-traditional strategies.

Prior research has documented various factors that support or inhibit PSTs' abilities to create effective mathematics instruction that would promote students' conceptual understanding. These factors include PSTs' content and pedagogical content knowledge (Ball, 1990; Sowder, 2007); beliefs about mathematics, mathematics teaching, and learning (Eisenhart