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Student noticing in classroom settings: A process underlying influences on prior ways of reasoning

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ABSTRACT

This study examines the degree to which individual and social aspects of student noticing in classroom settings during new learning influence students' ways of reasoning about previously-encountered concepts. Seventh- and eighth-grade students (N = 7) participated in an instructional unit on quadratic functions (the new concept) and clinical pre- and post-interviews examined students' ways of reasoning about linear functions (the previously-encountered concept). Qualitative analysis of the interview and classroom data revealed that (a) five of seven students' ways of reasoning on linear function tasks were productively influenced from pre- to post-interview, and (b) all seven students came to notice covariation during the quadratic functions instructional unit by way of particular social processes. Furthermore, the changes in ways of reasoning about linear functions were conceptually connected to what students noticed about quadratic functions. This study serves as a proof of concept that the process of noticing during new learning about quadratic functions can be leveraged to productively influence students' ways of reasoning about linear functions. This study could also serve as a model for how to enhance instruction for other mathematics topics to similarly achieve productive influences on ways of reasoning about previously-encountered concepts.

1. Introduction

Much theorizing and a large body of empirical research has been dedicated to identifying and characterizing the processes that underlie changes that knowledge undergoes after initial conceptions are formed (e.g., Chi, 2008; Karmiloff-Smith, 1992; Perry & Elder, 1997; Piaget, 1947/2002; Pirie & Kieren, 1994; Resnick, 1989; Smith, diSessa, & Roschelle, 1993; Vosniadou & Brewer, 1987; Wearne & Hiebert, 1988). Multiple processes, such as *accommodation* (Piaget, 1947/2002), *representational redescription* (Karmiloff-Smith, 1992), and *folding back* (Pirie & Kieren, 1994), have been used to account for *changes* to prior knowledge. The study reported in this article extends that research tradition by examining the degree to which the process of *noticing* during learning about new concepts can be used to account for *influences* on students' prior ways of reasoning (i.e., ways of reasoning about previously-encountered concepts).

Although *teacher noticing* has received considerable research attention (e.g., Jacobs, Lamb, & Philipp, 2010; van Es & Sherin, 2002, 2006), *student noticing* during classroom-based instruction has not. Exceptions to this rule are Lobato, Rhodehamel, and Hohensee (2012) and Lobato, Hohensee, and Rhodehamel (2013). Both articles provide a "dynamic and reflexive account of the contributions of both students and teachers to social interactions that are related to the emergence of what students

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notice" (Lobato et al., 2013; p. 813). For this study, noticing was defined as the process of mentally isolating particular features, creating/re-presenting mental records for those features and then, from those features, identifying particular regularities, properties, etc. (Lobato, Rhodehamel et al., 2012). Later in this article, this definition will be elaborated on.

The Lobato, Rhodehamel et al. (2012) study is particularly relevant to the current study because they reported that what students noticed in instructional settings had an *influence* on their subsequent reasoning on novel problems (i.e., problems that differed from the kinds of problems worked on during instruction). For example, students who noticed, during classroom-based instruction, that graph lines represent "collections of paired quantities that are related quantitatively" (p. 474), later on a novel task "[reasoned] with the quantities represented by the axes in the transfer task in order to find and interpret the slope in context" (p. 473). In contrast, students who, during instruction, primarily noticed the physical steepness of graph lines, later on the novel task, "[ignored] the quantities and [treated] graphs as constituted by boxes" (p. 473). In other words, what students noticed during instruction was associated with how students *generalized* their newly-constructed knowledge to novel problems.

Similar to Lobato, Rhodehamel et al. (2012), my study examined noticing as a mechanism for the generalization of knowledge. However, in contrast to Lobato et al., my study focused on influences from learning about new concepts on ways of reasoning about previously-encountered concepts, rather than on ways of reasoning about novel problems. Research has shown that learning about new concepts can generalize to ways of reasoning about previously-encountered concepts in productive ways (e.g., Arzi, Ben-Zvi, & Ganiel, 1985; Hohensee, 2014; Moore, 2012; Young, 2015) and unproductive ways (e.g., MacGregor & Stacey, 1997; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2004).

Based on the findings by Lobato, Rhodehamel et al. (2012) that noticing was linked to influences on reasoning about novel problems, I hypothesized that noticing may also be linked to influences by instruction about new concepts on ways of reasoning about previously-encountered concepts. From that hypothesis, the following research question emerged: *When and in what ways does the process of noticing underlie influences by instruction about new concepts (i.e., quadratic functions) on prior ways of reasoning about previously-encountered concepts (i.e., linear functions)*? Note that by focusing on underlying processes, I assumed a *process orientation to causality* (Maxwell, 2004). In other words, my goal was to examine whether noticing *connects* (not causes) new learning to influences on ways of reasoning about previously-encountered concepts (see the Methods for more on this orientation).

2. Theoretical foundation

In this section, a theoretical foundation for an investigation of noticing is developed. First, the framework that served as the foundation for this study on noticing is outlined. Second, the central problem that this study addressed is contextualized within this framework. Third, the mathematical context within which this study was situated is presented.

2.1. Framework for student noticing

The *focusing framework*, which was developed by Lobato, Rhodehamel et al. (2012) as a conceptual and methodological tool, was employed for this study as the perspective on student noticing. The focusing framework coordinates individual and social aspects of noticing, where the social component of the framework is Goodwin's (1994) *professional vision framework*, adapted and extended to mathematics learning.

2.1.1. Individual aspect of student noticing

According to the focusing framework, the individual aspect of noticing is a three-part process (Lobato, Rhodehamel et al., 2012). First, noticing involves mentally isolating particular features from a complex perceptual and/or conceptual field ("Focused attention picks a chunk of experience, isolates it from what came before and from what follows, and treats it as a closed entity," von Glasersfeld, 1995; p. 91). Second, noticing involves creating and re-presenting mental records of those selected features ("For the mind, then, 'to posit it as object against itself', is to re-present it," von Glasersfeld, p. 91). Third, noticing involves identifying regularities (or irregularities), properties, features, or conceptual objects from among those features ("establish[ing] regularities in the flow of experience," von Glasersfeld, p. 144). The focusing framework collectively refers to the regularities, properties, etc. that are identified as *centers of focus*.

My assumption for this study was that the individual aspect of noticing does not fully occur until centers of focus have emerged. In other words, I assumed that the first two parts of the noticing process were necessary but insufficient for noticing. Therefore, when I refer to noticing throughout this article, I mean that a center of focus has emerged. I also assumed that the individual aspect of noticing happens inside the mind and is not directly observable, particularly during classroom instruction (Lobato, Rhodehamel et al., 2012). Lobato et al. addressed this issue by using verbal utterances, written work and gestures produced by students during instruction to make inferences about centers of focus that emerged. I used a similar approach.

The centers of focus I was interested in for the study were those associated with the construction of mathematical knowledge. The constructivist perspective is one way to think about knowledge construction. According to my interpretation of constructivism, I see three ways that centers of focus may play a role in knowledge construction. First, centers of focus may play a role in *assimilation* because "in order to be activated, a scheme requires the perception of a particular pattern of sensory signals" (von Glasersfeld, 1995; p. 154). For example, suppose a student considers a quadratic function data table in