Journal of Mathematical Behavior 42 (2016) 109–126

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Representations of a mathematical model as a means of analysing growth phenomena

Carolina Guerrero-Ortiz [a,*], Hugo R. Mejía-Velasco [b], Matías Camacho-Machín [c]

[a] Pontificia Universidad Católica de Valparaíso, Chile
[b] Centro de Investigación y de estudios Avanzados de Instituto Politécnico Nacional, Mexico
[c] Universidad de La Laguna, Spain

ARTICLE INFO

Article history:
Received 17 August 2015
Received in revised form 26 February 2016
Accepted 8 March 2016
Available online 19 April 2016

Keywords:
Digital tools
Interpreting
Ordinary differential equations
Representations
Mathematical models

ABSTRACT

In the context of qualitative research, this paper presents and analyses the strategies students used and the mathematical ideas that come to light when working with two tasks involving the study of relationships arising between a contextual situation and an associated mathematical model. The results show that by studying the different mathematical representations with the support of digital tools, it is possible for students to develop different analysis strategies that help them to understand the behaviour of the phenomenon and the connections between the associated representations. Extensive use of registers of representation could be seen as a means of interpreting and establishing the relationship between the mathematical model and the corresponding context. These results are an extension of the work on the qualitative study of differential equations and how students interpret them.

1. Introduction

Several research studies discuss the problem of teaching and learning Differential Equations (DE) (Arslan, 2010a, 2010b; Rasmussen, 2002; Guerrero-Ortiz, Camacho-Machín & Mejía-Velasco, 2010; Camacho-Machín & Guerrero-Ortiz, 2015), particularly in reference to the obstacles to learning generated by prioritising the teaching of algorithmic strategies over the development of conceptual reasoning strategies, thus encouraging students to memorise theorems, definitions, rules and algorithms, rather than comprehend the mathematical concepts and relations that can emerge by restructuring prior knowledge (Arslan, 2010b). Therefore, the knowledge and skills acquired by students are mediated by the nature of the mathematical activities performed in the classroom. Many efforts have been made to introduce numerical and geometrical approaches to support the study of differential equations (Artigue & Gautheron, 1983, Artigue, 1989; Hernández Ramírez, 1995; Rasmussen & Kwon, 2007), but few results have been seen in the classroom. In order to contribute to the knowledge developed in this direction, the aim of the present study is to show how the use of different semiotic representations associated with ODEs can appear naturally when students work with tasks where they have to determine how a phenomenon develops or give meaning to the mathematical models associated with it. A key element was the students' effective use of digital tools.

* Corresponding author.
E-mail address: c_cguerrero@yahoo.com.mx (C. Guerrero-Ortiz).

http://dx.doi.org/10.1016/j.jmathb.2016.03.001

With regard to the learning of differential equations, Rasmussen (2002) argues that two themes can be analysed: the function as a solution and the students' intuitions and images. He pays special attention to interpreting solutions, interpreting equilibrium solutions, focusing on quantities, numerical approximations and stability. Rasmussen warns that working with the notion that DE solutions are functions may not be an easy task for students since much of their past experience is with solving equations where the solutions are numbers. Therefore, the problem regarding the concept of function is transferred to the context of differential equations. Rasmussen also reports that when students work with equilibrium solutions they appear to develop an intuitive theory about the existence of equilibrium solutions. In other words, students tend to think that equilibrium solutions exist whenever the derivative is zero and associate the stability with consistency or regularity in solutions. Rasmussen observed some difficulties that students have in identifying the relevant information presented in graphs as they pay attention to amounts that are not relevant.

Students find it difficult when they try to give meaning to functions or differential equations when these represent a phenomenon or situation (Rowland & Jovanoski, 2004; Guerrero-Ortiz, Camacho-Machín & Mejía-Velasco, 2010; Nemirovsky & Rubin, 1992). This challenge requires research that takes into account the diversity of exploration routes offered by different semiotic representations and the importance of interpreting the information given by a mathematical model, in this case Ordinary Differential Equations (ODE). The present research was developed in the context of the design and implementation of a set of teaching activities aimed at developing students' abilities to interpret the elements of algebraic expressions of an ODE with particular focus on equations modeling population growth. Some of the strategies used by students when building and exploring mathematical models originating from the study of growth phenomena will be shown. The analysis also looks at how the students use digital tools to mobilise their resources to associate the context of a problem with the representations that arise while they construct and explore the mathematical models (ODE) describing the phenomena in question. In this context, questions that come up are: *What strategies do students use to understand a phenomenon? How do they represent a phenomenon mathematically? Are they able to make connections between mathematical representations associated with a phenomenon? How? What is the role of software in all this?* Answering these questions will highlight the extent to which manipulation of different representations of the modelled situation is a valid mean of analysing and developing meaning with respect to the elements that are used to build an ODE.

2. Conceptual framework

The study of differential equations can be approached from several perspectives to shed light on the presence of different representations, both of the equation and of its solutions. In particular, in the study of the behaviour of some phenomena, algebraic representations arise when the aim is to comprehend how the phenomenon in question develops by studying the mathematical model associated with it, though the solutions cannot always be found algebraically. In this context and with the support of digital tools, numerical and graphical methods acquire relevance for the exploration of some types of differential equations by studying their qualities and the relationships between the different representations of the mathematical object. These representations acquire a role of similar importance in the study of Systems of Differential Equations as reported by Dana-Picard & Kidron (2008) and Rasmussen & Blumenfeld (2007). Thus, the nature of the study of DEs and systems of DEs suggests the suitability of analysing the role of the different representations in the students' conceptualisation when beginning to study these subjects.

As mentioned by Duval (1997), mathematical objects are not directly accessible to perception and it is only possible to access them through their semiotic representations He says that:

> "Mathematical objects, in contrast to phenomena of astronomy, physics, chemistry, biology, etc., are never accessible by perception or by instruments (microscopes, telescopes, measurement apparatus). The only way to have access to them and deal with them is using signs and semiotic representations. That means that we have here only a single access to the knowledge objects and not a double access, mainly non-semiotic and secondarily semiotic, as is the case in the other areas" (Duval, 2006)

Duval states that the comprehension of mathematical objects can only be conceptual, and that only by means of their representations is it possible to perform an action on these objects. He also distinguishes between mental representation and semiotic representation. Mental representations comprise the set of images and conceptions that an individual has with regard to an object or situation. Semiotic representations can be considered as a means of externalising mental representations. Three cognitive activities (register formation, treatment in the same register and conversion between registers) identified by Duval are related with semiotic representations. In (Camacho-Machín & Guerrero-Ortiz, 2015) paraphrasing Duval we summarised:

> "...producing an answer in a mathematical activity involves the formation of semiotic representations and their simultaneous treatment, whereas the understanding of a concept requires the cognitive activities involved in register formation and in conversion between registers, or the simultaneous application of the three cognitive activities. Duval said that the mobilization of several representation registers was necessary to avoid confusing mathematical objects with their representations. (Camacho-Machín & Guerrero-Ortiz, 2015)"