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Students' conceptions of reflection: Opportunities for making connections with perpendicular bisector

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ABSTRACT

Given the current emphasis on the use of transformations for the teaching and learning of geometry, there is opportunity to consider how students' understanding of geometric transformations can be used to build connections with interrelated concepts. We designed a sequence of three problems, collectively referred to as "the pottery lesson," to elicit evidence of students' understanding of reflections. We asked: *What conceptions of reflection did students use while working on the pottery lesson? How did students' work on a sequence of problems requiring reflecting create opportunities for establishing connections between reflections and perpendicular bisector?* We identified opportunities for the use of perpendicular bisector to shift between an operation of students' work and a measure of control. The characterization of students' conceptions of reflection, and students' related use of perpendicular bisector, provide a resource for the teaching of these concepts to build upon students' prior knowledge to promote learning.

In the United States, the *Common Core State Standards for Mathematics* (National Governor's Association Center for Best Practices & Council of Chief State School Officers [NGAC], 2010) place a renewed emphasis on the use of geometric transformations for establishing relationships between geometric objects (NGAC, 2010). In particular, students should have opportunities to develop a definition of a reflection as a transformation using perpendicular lines and segments (NGAC, 2010, p. 76). With the opportunity to revisit the use of transformations in geometry classes, it is necessary to know how to leverage students' prior knowledge of reflections for the purpose of making its connection with perpendicular bisector explicit. Prior research has documented the different ways that students reflect figures (Mhlolo & Schäfer, 2014), discuss reflective symmetry (Hoyles & Healy, 1997), and prove conjectures related to reflections (Miyakawa, 2004). We seek to expand research on students' understanding of reflections by examining how students' work on problems about reflecting provides opportunities for a teacher to establish connections with the perpendicular bisector. This work considers how teachers can use the emphasis on transformations in geometry to build connections with interrelated objects and relationships.

We are interested in the question, how do students' conceptions of reflective symmetry either highlight or obscure connections with perpendicular bisector? We designed a sequence of problems to elicit students' prior knowledge of reflections and to provoke the use of perpendicular bisector in relation to reflections. Through an investigation of students' work, we propose how an instructional sequence may build upon students' initial conceptions to promote students' ability to

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apply perpendicular bisector in relation to reflections. With our overarching question, we seek to contribute to a broader investigation of how teachers can use students' prior knowledge to establish connections between mathematical ideas.

1. Research questions

We examined students' work on a sequence of problems collectively referred to as "the pottery lesson," asking two research questions: (a) What conceptions of reflection did students use to complete the pottery lesson? (b) How did students' work on a sequence of problems requiring reflecting create opportunities for establishing connections between reflections and perpendicular bisector? We used students' work on the pottery lesson to consider how instruction can leverage students' prior knowledge to foster connections between perpendicular bisector and reflective symmetry.

2. Conceptual framework

We based our analysis on the cK¢ (pronounced "c-k-c") model of students' conceptions (Balacheff, 2013; Balacheff & Gaudin, 2002, 2010). The cK¢ model, which stands for conception-knowing-concept, makes a distinction between mathematical concepts and the different conceptions that students may have about a single concept. The cK¢ model of conceptions uses a definition of a mathematical concept as "a set of situations, a set of operational invariants (contained in schemes), and a set of linguistic and symbolic representations" that give meaning to a particular idea (Vergnaud, 2009; p. 94). The concept of reflection, for example, encompasses multiple problem situations, such as a problem about constructing a reflection or a problem about proving that a reflection satisfies certain properties. We also consider here that the concept of reflection, which emphasizes the use of a transformation, can be considered distinct from the concept of reflective symmetry, which emphasizes a property of a single figure.[1] Because different types of problems elicit different uses of a concept, the definition of concept is meant to encompass all of those different uses, including different actions and representations one might use to solve the problems.

The term *students*' *conceptions* has been used in the past to refer generally to students' explanations of mathematical concepts, students' beliefs, "naïve theory," or "the mathematics of the child" (Confrey, 1990). Students may have multiple ways of using a concept such as reflection to solve problems in mathematics. A reflection, as a geometric transformation, specifies a relationship between two points or figures. The process of reflecting refers to the activity involved in producing such a relationship. There are multiple ways that students may solve problems about creating or describing reflections, for example through the use of drawings, geometric constructions, or measurements. Given these differences, it is conceivable that students have multiple conceptions of reflection, which brings to light the importance of characterizing these different conceptions. The cK¢ model operationalizes the construct of a conception to allow for examination of students' understanding of a concept through their work in a particular context.

2.1. The role of a problem in a conception

The cK¢ model of students' conceptions builds upon the theory of conceptual fields (Vergnaud, 1981, 2009) and the theory of didactical situations (Brousseau, 1997), which provide a background for understanding how mathematics problems become part of the ways that students think about mathematics. We use the term *learner* to refer to a student who is engaging with a particular mathematics concept (Balacheff, 2013; Brousseau, 1997). The *milieu* is the subset of the learner's environment that is relevant for learning that particular concept. For the purpose of studying students' conceptions, we consider students as learners in relation to the knowledge, representations, tools, and questions relating to some geometric concept. The milieu can be thought of as the learner's "antagonist system" to provoke the learning process (Brousseau, 1997; p. 57). Learners interact with the milieu by posing questions and developing and implementing strategies to solve problems with the resources available. The milieu provides feedback to the student. For example, given a pre-image and a line of reflection drawn on a piece of paper, a student might reflect the pre-image by folding the paper along the given line and tracing. The student uses aspects of the milieu – namely drawing upon the knowledge of what a reflection looks like, folding the paper, and tracing the image – to determine whether she or he has drawn the reflection correctly. The student gets feedback from the milieu in the form of whether the resulting figure looks like a reflection, or whether the pre-image and image align across the line of reflection.

A mathematical conception emerges through the interaction between the learner and the milieu. A *problem* is a perturbation in the learner-milieu interaction, which the learner must resolve (Balacheff, 2013). For example, suppose a student is asked to create a figure with reflective symmetry. The student would not have the option of using a given line of reflection and tracing a given figure, a method that may have previously worked on another problem. The student might first draw a line of reflection and then construct points equidistant from that line of reflection. Alternatively, the student could draw a collection of points on paper, fold the paper, and trace to create a single image with reflective symmetry. In this way, problems can be thought of as the source of a learner's understanding of a particular concept (Vergnaud, 1981). Problems

[1] Reflection and reflective symmetry can, however, be considered elements of a single conceptual field, which describes a collection of interrelated concepts and problem situations (Vergnaud, 2009).