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Middle school students' patterning performance on semi-free generalization tasks

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ABSTRACT

This longitudinal study empirically addresses the issue of structure construction and justification among a class of US seventh and eighth-grade Algebra 1 students (mean age of 12.5 years) in the context of novel semi-free pattern generalization (PG) tasks before and after a teaching experiment that emphasized a multiplicative thinking approach to patterns. We compared the students' PG responses before and after the experiment and found that (1) one source of variability in their abduced structural processing was in part due to an initial conceptual preference toward thinking either in parts or in wholes and (2) a multiplicative understanding of structures significantly aided them in PG conversion (e.g., from the visual to the alphanumeric) and processing (e.g., from nonstandard to standard function-based formulas). Our findings provide both necessary and sufficient conditions for constructing, establishing, and justifying valid structures in the case of (semi-) free figural patterning tasks.

1. Introduction

Children and adults have been documented as having a natural disposition toward pattern generalization ("PG;" Cadez & Kolar, 2015; Jurdak & El Mouhayar, 2014; Nilsson & Juter, 2011; Rivera, 2013; Tanisli & Özdas, 2009; Walkowiak, 2014; Wilkie, 2016). PG is an ability that involves constructing and justifying a well-defined mathematical structure for a given set of initial cues or particular cases. For example, Fig. 1 is a figural pattern with four given stages *and* with no additional information other than what individual learners infer about its structure that will then be applied to the given stages and beyond.

A *mathematical structure* refers to "a mental construct that satisfies a collection of explicit formal rules on which mathematical reasoning can be carried out" (National Research Council, 2013; p. 29). Hence, PG and mathematical structure are intimately and conceptually intertwined, meaning to say that PG ability is interpretive and rule-driven in nature and enables learners to employ predictive and inferential reasoning despite the initial constraint of having only an incomplete knowledge of the target objects for generalization (e.g., stages in a pattern such as those shown in Fig. 1 or a set of particular instances, situations, or cases). Consequently, every pattern has to be *well-defined*, that is, any stage, cue, or instance must either exhibit the structure or not exhibit the structure. For example, Fig. 2 shows three different but equivalent student-generated structures for the Square Tiles Pattern in Fig. 1.

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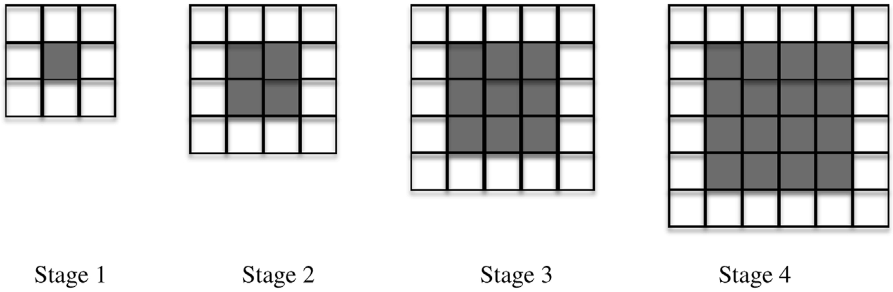
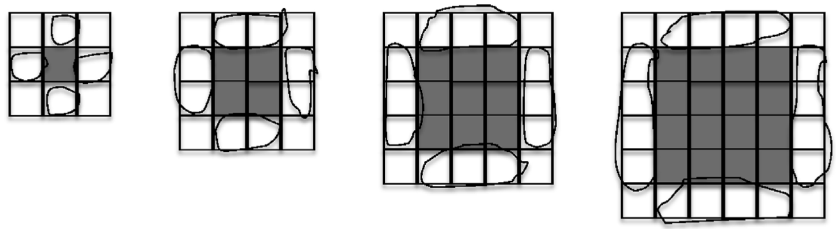
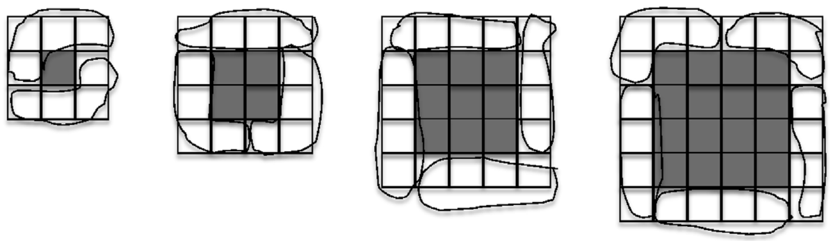


Fig. 1. Square tiles pattern.

Che (age 12 years): "W = 4n + 4. So there's like 4 squares and then you add 4 to each one [corner]. … And for patio #2, 2 x 4 is 8, so 1, 2, 3, …, 8, then you add 4. [There are] 4 groups of n."



Dina (age 12 years): "4(n + 1) coz patio #1 you have two groups of 4, patio #2, 3 groups of 4, etc."



Dave (age 12 years): "T = 2(n+2) + 2n. The top part, 2 + 1 = 3. Then I multiplied by 2, the bottom, so that's 6. And the 2n, so here's 1 [row 2 column 1 square] and 1 here [row 2 column 3 square], etc."

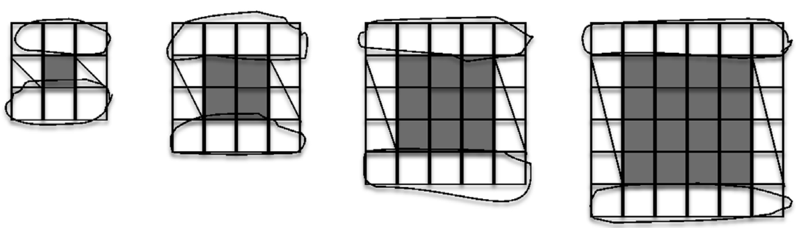


Fig. 2. Grade 8 student responses on Fig. 1 pattern.

On the basis of eighth-grade student Dave's articulated structure, it is easy to infer and predict that stage 235 will have, say, two (top and bottom) rows of 237 unshaded tiles and two (left and right) columns of 235 unshaded tiles surrounding a 235×235 shaded square. Furthermore, the students' combined verbal and alphanumeric responses in Fig. 2 convey their respective *intensional generalizations* for the Fig. 1 pattern, that is, depth-driven statements that express how they analytically visualize the pattern. In Duval (2014) terms, this visual process is mathematical and involves discerning and recognizing