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Folding back, thickening and mathematical met-befores

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ABSTRACT

The study reported here explores how teachers can encourage students to build mathematical understandings that are connected to their prior experiences and knowledge. We illustrate this through considering data extracts taken from the classroom of one high school teacher, as he works with a Grade 12 class on the concept of vectors in three-dimensional space. Drawing on elements of the Pirie-Kieren theory for the dynamical growth of mathematical understanding we consider how the deliberate and purposeful teaching action of encouraging folding back can offer a way for students to engage with problematic mathematical met-befores and so promote understandings that are both more general in nature and conceptually connected.

1. Introduction

The NCTM 'Learning Principle' (NCTM, 2000), that focuses on the importance of conceptual understanding, states "students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (p. 20). However, it does not speak directly to specific pedagogical actions or teaching strategies that might occasion such building, and it is the identification of these on which this paper focuses. In particular we consider how the deliberate and purposeful teaching action of encouraging "folding back" with its associated cognitive act of "thickening" (Martin, 2008) can offer a way for students to engage with specific "problematic met-befores"—prior knowings that are insufficient for a new context (McGowen & Tall, 2010) —and so promote understandings that are both more general in nature and conceptually connected.

2. Prior knowings and mathematical met-befores

The importance of prior knowledge in the process of learning mathematics is well acknowledged (see Campbell & Campbell, 2009; Hatano, 1996; Mack, 2001). As noted above, the NCTM Learning Principle (NCTM, 2000) emphasizes building new knowledge from prior knowledge, and Krainer (2004) states that "taking students' prior knowledge into account is a prominent goal in mathematics education" (p. 87). More specifically An, Kulm, and Wu (2004) state that "using prior knowledge not only helps students to review and reinforce the knowledge being taught, but also helps them to picture mathematics as an integrated whole rather than as separate knowledge" (p. 165).

However, there remains limited research on what "using prior knowledge" (or, as also described in the literature, accessing; building on; invoking; relating to; connecting; reviewing; reinforcing; solidifying; integrating; activating; tapping into,

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etc.) might involve and on the potential role of the teacher in enabling this kind of process. As Mack (2001) asks “how might students actually return to their initial understandings, and in what ways may this return influence the development of their understanding of the domain? Insights into these issues are not yet clear” (p. 269). She also notes that “students may not always return to their initial understandings on their own even when this return would be beneficial. Students may require guidance that leads them to reconsider these understandings at opportune times for growth to occur” (p. 269). In a similar vein Krainer (2004) argues that

> taking students prior knowledge seriously means building up on the existing ideas and strengths of students, picking them up where they are. This means neither ignoring the goals that should be aimed at nor abandoning our high expectations. However, it does mean that students’ learning cannot be reduced to a comparison of their knowledge with expert knowledge and hence only observed and assessed in terms of errors and misconceptions. Instead, we need to focus very intensively on students’ growing understanding (p. 88).

In considering the role that prior knowings and experiences might play in new learning, and also highlighting the risk of reducing prior knowledge to errors and misconceptions, McGowen and Tall (2010, 2013) introduced the notion of “met-befores”. Originally a word play on metaphor, McGowen and Tall (2010) define met-befores as “all current knowledge that arises through previous experience, both positive and negative” and offer a working definition as “a mental structure that we have *now* as a result of experiences we have met-before” (p. 171). They contrast met-befores with the well-established notion of epistemological obstacles (Brousseau, 1983; Sierpinska, 1994) noting that met-befores can both support as well as prevent the acquisition of new knowledge, but that an awareness of the importance of met-befores, by both teachers and learners, is vital. In particular they draw attention to the way that teaching is often structured through deliberately “teaching pre-requisites that are required in a supportive role in later learning. Less often is the role of problematic met-befores made explicit. If they are discussed at all, it is often in the role of ‘misconceptions’ and ‘misunderstandings’” (McGowen & Tall, 2010, p. 172). Of particular significance for this paper is their recognition that without “addressing the problematic met-befores that remain under the surface any chance of conceptual understanding is suppressed and the only way forward is to focus the student on the techniques required to succeed in performing correct methods of getting the answer” (McGowen & Tall, 2010; p. 172).

What McGowen and Tall (2010) do not do though is offer particular descriptions of what “addressing” met-befores might mean, or of how teachers might facilitate this. In this paper, we draw on the theoretical metaphor of ‘folding back’ (Martin, 2008), and consider how this might offer a way for teachers to explicitly create opportunities for learners to engage with and address their mathematical met-befores.

3. Folding back and the dynamical growth of mathematical understanding

The research reported in this paper is framed by the Pirie-Kieren theory for the dynamical growth of mathematical understanding (Pirie & Kieren, 1989, 1992, 1994). The theory views mathematical understanding not as a static state to be reached or achieved, but as a dynamical, growing, and ever-changing process. Hence, it becomes appropriate to talk not about ‘understanding’ as such, but about the dynamical process of coming to understand and about the ways that mathematical understanding shifts, develops, and grows as learners move within a mathematical world. Kieren, Pirie, & Gordon-Calvert, 1999 talk of “understanding in action” as being “embodied and. . .in that embodiment the more formal and abstract actions unfold from, but are connected to, less formal actions” (p.212). Such a view also recognizes that “because growth of understanding occurs in contexts, a study of the growth of understanding must necessarily take into account the interactions that a person has with and in such contexts, including interactions with materials, other students and teachers” (Kieren et al., p. 229).

The Pirie-Kieren theory posits eight nested layers of understanding (illustrated as a diagrammatic representation or model in Fig. 1) to describe ways in which a learner can be observed to act mathematically, and characterises the growth of mathematical understanding as emerging through the continual movement back and forth through the layers of knowing, as individuals reflect on and reconstruct their current understandings.

The nesting of the layers illustrates the fact that growth in understanding need be neither linear nor mono-directional. In addition, each layer contains all previous layers and is included in all subsequent layers, to emphasize the embedded nature of mathematical understanding. Using the model, the growth of understanding of a learner, for a particular mathematical concept, can be mapped out. Fig. 1 illustrates a diagrammatic representation of a hypothetical pathway of the plotted growth of understanding of a particular mathematical concept, for a learner or group of learners, over a particular period of time. This choice of focus and time-scale is fundamental to the use of the theory, and is one made by the observer.

The definitions of the various layers of understanding actions have been fully set out in earlier work by Pirie and Kieren (Pirie & Kieren 1989, 1992, 1994) and thus here only brief definitions of the layers relevant to the data analysed in this paper are offered (that is, those layers at which understanding actions were observed in the analysis of the data). Primitive Knowing is seen as the starting place for the growth of understanding of any piece of mathematics. Primitive Knowing is observed to be everything that a learner knows (and can do) except the knowledge about the particular concept that is being considered by the observer. Anything that the learner may already know about that specific concept is seen, through the lens of the Pirie-Kieren Theory, to be an understanding on one of the other outer layers. At Image Making the learner is engaging in activities aimed at helping him or her to developing particular representations for the topic and mathematical idea; to