Journal of Mathematical Behavior 41 (2016) 219–234

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Truth isn't everything: Promoting aesthetically guided choice in mathematical work

Nicholas Fiori[a,*], Sarah Kate Selling[b]

[a] Saint Ann's School, United States
[b] University of Michigan, United States

ARTICLE INFO

Article history:
Received 19 July 2014
Received in revised form 3 September 2015
Accepted 6 October 2015
Available online 26 November 2015

Keywords:
Aesthetics
Student choice
Problem posing
Mathematical practices
Agency

ABSTRACT

Most educational and philosophical thought about mathematics focuses on the logical structure of the subject and considers mathematicians and students to be people whose primary practices are verifying statements within this structure. We claim that acts of discernment – careful choices driven by aesthetic considerations – are as important as acts of verification in mathematical work. This paper offers a conceptualization of this "aesthetically guided choice" that differentiates between three interrelated acts of discernment: nominating ideas, arranging ideas, and balancing ideas. We argue that aesthetically guided choice should be supported in school, and that such acts are notably lacking from the "Standards for Mathematical Practice" in the Common Core State Standards. This paper is the development of a theory at heart, built around rich descriptions of a mathematics class with middle school students, in which young thinkers were engaged in aesthetic choice. It includes analyses of the history of mathematics and studies of mathematical work to support claims about the nature of mathematics and to interpret the work of young mathematics learners from the perspective of mathematics as a discipline.

© 2015 Elsevier Inc. All rights reserved.

Contrary to popular belief, mathematicians spend very little time performing numerical computations or rigorously proving results. More often they face the much more daunting task of finding good ideas from the myriad possibilities. Posing an interesting idea or problem is an especially difficult challenge; a truly great problem or argument is the rarest of finds. Searching for these gems is a messy, logically incoherent process where misconceptions lead to progress just as often as correct deductions.[1] Students, on the other hand, typically have their days and evenings filled with neat computational tasks (Stigler & Hiebert, 2009). Rarely are they entrusted with the much more open-ended task of selecting ideas or negotiating what makes a worthwhile idea. This paper considers a school environment in which students were supported in these discerning acts, and develops an argument that such participation is an important, but often missing, part of mathematical learning. The argument is informed and illustrated with examples of middle school students engaged in making such mathematical choices in a learning environment designed to promote such acts. We then interpret the students' engagement in these acts of discernment through the lens of the practice of professional mathematicians as examined through historical documents and interview studies.

* Corresponding author at: Saint Ann's School, 129 Pierrepont St., Brooklyn, NY 11201, United States.
E-mail address: nfiori@saintannsny.org (N. Fiori).

[1] These claims about the nature of mathematics are discussed more in the opening section, and are a conclusion of a literature review of mathematical practices in Fiori (2007).

http://dx.doi.org/10.1016/j.jmathb.2015.10.002
0732-3123/© 2015 Elsevier Inc. All rights reserved.

The mathematics classroom we designed and studied was part of the summer school algebra teaching intervention discussed in this special issue (see Boaler, in this issue). This classroom was designed to be a more extreme take on the themes of our intervention, as students played an even more major role in making major decisions about content. Like in the other classrooms, a broad spectrum of mathematical practices was promoted, but there was an additional and significant emphasis on engaging students posing their own problems and making creative, aesthetically guided choices about what to study.

1. A key mathematical practice is missing from the lexicon

Descriptions of mathematical work have livened up a bit in recent years. While mathematical work was once described with such passive verbs as 'plug in,' calculate, and simplify, a new set of actions is making its way into the lexicon. Math educators now emphasize that a math student should have the opportunity to represent, justify, explain, argue, conjecture, and understand (e.g. Boaler, 2009; Bowers, Cobb, & McClain, 1999; Brown, Collins, & Duguid, 1989; Yackel & Cobb, 1996). With these richer descriptions, the practice of mathematics is portrayed more vibrantly and authentically. Beginning with the NCTM standards in 1989 and later refined in 2000, mathematics educators have worked to lay out the processes that students should be engaged in while learning and doing mathematics, such as problem solving, reasoning and proof, communication, making connections, and representation. More recently, the Common Core State Standards NGACBP (2010) have built on the NCTM process standards to further articulate the "mathematical practices" that should serve as the actions through which students access, learn, and use the mathematical content. The eight standards for mathematical practice are meant to represent how students should be actively engaging in mathematics across all grade levels (K-12) and content domains.

Despite these valuable developments, conceptions of the discipline are still found wanting. Descriptions of participatory actions in mathematics tend to focus on logical, propositional thinking. While these activities are no doubt an important component of mathematics, people who are immersed in the subject rely on another essential action. Analyses of the history of mathematics and studies of mathematicians doing their work reveal that *acts of discernment* are as important as *acts of verification* (Fiori, 2007[2]). No matter who is the mathematician – Archimedes, Évariste Galois, Maryam Mirzakhani or a sixth grade student posing a problem for the first time – aesthetically guided acts of judgment play as important a role in the progress of mathematical work as do the chains of propositional reasoning more often associated with the subject. Not only are mathematicians drawn to the subject and find meaning in their work because of its aesthetic appeal, but practicing these sensibilities makes their work possible. Hy Bass (2011), a research mathematician and math educator, reflects meta-cognitively on the mathematical practices he uses while producing new mathematics. In addition to practices such as exploring, constructing proofs, and making connections, Bass's list of practices highlights asking "natural questions" and being guided by "aesthetics and taste" (2011, pp. 4–7). Bass writes, "As in any profession, mathematicians are diverse in their styles and tastes. Still, in mathematics, there is a remarkable degree of shared aesthetic sensibility – associated with words like elegance, precision, lucidity, coherence, unity, . . . – that affects not only how they appreciate, but even how they do mathematics" (p. 7). Mathematicians, like visual artists and writers, are guided by judgments of taste, or aesthetic choice, and seek ideas that have an element of beauty. They can recognize what they consider to be a 'good' problem as readily as a painter notices a significant work, and they can tell you why they like it. Part of what makes the subject so striking is that the beautiful choice very often turns out to be the most sensible.

Accordingly, aesthetically grounded views of mathematics offer an important perspective for education. In addition to asking whether or not our students understand mathematics, we should be asking whether or not they are supported in making judgments about mathematics. Do our students leave school with a developed mathematical taste? Do they have favorite problems or ideas? Both the value and shortage of students being encouraged to pose and refine mathematical problems has been articulated by small set of researchers (Brown & Walter, 2005; English, 1998; Silver, 1994, 2013). An even smaller fraction has defended the importance of aesthetic choices more generally (King, 1993; Sinclair et al., 2004; Sinclair, 2006; Tymoczko, 1993). Of these researchers, those who promote such practices in school wisely tend to do so within the context of existing topics of study. We believe it is also of research value to explore other environments that more closely approximate professional research environments, whether or not the questions and topics stray from standard school curriculum. This gives students greater freedom to engage fully in the process of making aesthetic judgments in mathematics. We argue in this paper that it is crucial for the advancement of research knowledge that we create and examine environments where students are engaged in making aesthetically guided choices in mathematics, whether or not we can yet see exactly how these practices could be integrated into a more standard school day and curriculum.

2. An urgent problem

Students, as well as many parents and teachers, often tell us that even though they work hard at school mathematics, they feel their efforts lead toward somewhat vacant ends. Interviews and assessments of students tend to show students leave math with an absence of perspective, larger meaning, or personal investment in the subject (McLeod, 1989). Math students

[2] In Fiori (2007) I offer this analysis, drawing from works such as: Hadamard (1945), Halmos (1968), Tymoczko (1993), Burton (1999), Netz (2000, 2003), Stillwell (2006), and others.

Download English Version:

https://daneshyari.com/en/article/360625

Download Persian Version:

https://daneshyari.com/article/360625

Daneshyari.com