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Levels of participatory conception of fractional quantity along a purposefully sequenced series of equal sharing tasks: Stu's trajectory

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ABSTRACT

Current intervention research in special education focuses on children's responsiveness to teacher modeled strategies and not conceptual development within children's thinking. As a result, there is a need for research that provides a characterization of key understandings (KUs) of fractional quantity evidenced by children with learning disabilities (LD) and how growth of conceptual knowledge may occur within these children's mathematical activity. This case study extends current literature by presenting KUs of fractional quantity, evidenced through problem solving strategies, observable operations, and naming/quantification of one fifth grader with LD before, during, and after seven instructional sessions situated in equal sharing. The researchers utilized a characterization of evolving fraction conceptions developed from research of children without disabilities that was ultimately productive in facilitating conceptual advances of the child with LD. We hypothesize that the trajectory of the child's conceptions is a case of something more general. Pending future research, the trajectory may be a useful tool to practitioners wishing to plan thoughtful, conceptually-based fraction instruction that is responsive to *all* children's evolving conceptions of fractions as quantities built through their own mathematical activity.

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1. Introduction

Elementary children labeled as having learning disabilities (LD) begin their study of fractions with similar yet more diminished conceptual understandings of fractions as quantities then what is documented among their peers without disabilities (Hunt & Empson, 2014; Hecht, Vagi, & Torgesen, 2007). An incomplete understanding of fraction concepts impacts children's ability to operate with or apply computational procedures with fractions in higher-level mathematical contexts (Hecht & Vagi, 2010; National Mathematics Advisory Panel, 2008). These findings suggest a continued instructional focus on the development of conceptual understanding of fractions as quantities for these children is critical.

Much of the intervention literature in special education is based on an interpretation of the child as *deficient* (e.g., Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Fuchs et al., 2014; Test & Ellis, 2005). Consequently, interventions employ explicit,

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systematic instruction to deliver knowledge onto children with LD and measure their subsequent 'responsiveness' to instruction (Butler et al., 2003; Fuchs et al., 2014; Test & Ellis, 2005; Zhang, Steckler, Huckabee, & Miller, in press). Features of the instructional design typically include (a) teacher-directed learning (i.e., ownership or modeling of thinking) when new ideas are introduced, with children's restatement of that thinking emphasized, (b) specific strategies, models, representations, and vocabulary modeled by the teacher to be used by children to learn new material, (c) a purposeful review of previously mastered content, and (d) multiple opportunities for children to practice the teacher's demonstrated ideas. The focus on explicit instruction is due in part to prior research in the field of special education that suggests children labeled LD do not benefit from constructivist-based forms of instruction (e.g., Baxter, Woodward, & Olson, 2001; Baxter, Woodward, Voorhies, & Wong, 2002).

Explicit instruction seems beneficial if the goal of research and practice is to increase children's efficiency with teacher-demonstrated strategies or procedures, or memorization of mathematical conventions (Hiebert & Grouws, 2007). Yet, if conceptual understanding is the goal of instruction, then it cannot be imposed on children (Baroody, Cibulskis, Lai, & Li, 2004). Instead, supporting children's conceptual growth involves *the child* "[solving] problems within [their] reach [while] grappling with key mathematical ideas that are comprehendible but not yet well formed" (Hiebert & Grouws, 2007, pp. 387). Thus, the child is not deficient; he is a capable, active learner who builds conceptions based within his informal knowledge and mathematical activity when immersed in a series of thoughtful tasks. If teachers view the child with LD as capable and wish to use instruction to facilitate conceptual growth, then a characterization of the understandings he *does* have of fraction concepts and how growth of conceptual knowledge might occur may provide an indispensable foundation. Unfortunately, no such characterization currently exists in the literature.

One way to provide the needed characterization is through the construction of children with LD's trajectory of learning (Daro, Mosher, & Corcoran, 2011; Simon, 1995). Trajectories model key understandings (KUs), or critical transitions, in how a child conceives of a mathematical idea while immersed in a series of carefully sequenced tasks that elicit cognitive dissonance for the child (Simon, 1995, 2006). They illustrate how a child's existing notions of mathematical ideas may be elicited, the grappling of ideas a student might experience, and how more solidified notions of mathematics form through *the child's* internal mental activity; actions strategies for problem solving. Many different trajectories of how fractional knowledge might come about exist in the literature for children without disabilities (e.g., Empson, 1999, 2003; Steffe & Olive, 2010; Tzur, 1999). Yet, although it is reasonable to base *initial* trajectories of children with LDs conception on those of children without LDs, we cannot assume that *all* variants within existing trajectories will align with children with LD's conceptions. It is possible these children may have different ways of solving problems and/or utilizing cognitive structures than one might expect when teaching fraction concepts (Geary, Hoard, & Nugent, 2012).

In the following paragraphs, we introduce a two part theoretical framework used in the current study. The first part of the framework is comprised of varying KUs[1] involved with children understanding fractions as quantities gleaned from a synopsis of previous research (Charles & Nason, 2000; Empson, Junk, Dominguez, & Turner, 2005; Empson & Levi, 2011; Kieren, 1976; Piaget, Inhelder, & Szeminska, 1960; Streefland, 1993; Tzur, 1999; Steffe & Olive, 2010). The second piece of the framework is comprised of an articulation of the mechanisms that children rely on that promote development within their own mathematical activity (Simon et al., 2010; Simon, Tzur, Heinz, & Kinzel, 2004; Tzur & Simon, 2004). Together, the hypothesized trajectory of children's thinking and the mechanisms hypothesized to drive its evolution give rise to our research questions, which we introduce at the conclusion of the theoretical framework.

2. Conceptual framework

2.1. Key understandings in equal sharing tasks

Children's work within equal sharing tasks – equally sharing a given number of objects among different numbers of people, where the result is a fractional quantity – can evoke a variety of strategies, operations, and language reflective of varying KUs (Charles & Nason, 2000; Empson et al., 2005; Empson & Levi, 2011; Hunting & Sharpley, 1988; Kieren, 1976; Steffe & Olive, 2010; Pitkethly & Hunting, 1996; Steffe, 2002; Streefland, 1993; Tzur, 2007). We use the problem of sharing three clay bars equally among four people as an illustration; KUs outlined in previous research are described and italicized. In the most basic strategies for such problems, children may see the situation as unsolvable; *the whole, for them, is not yet divisible* beyond rudimentary notions of one-half (Empson et al., 2005; Empson & Levi, 2011; Piaget et al., 1960). In this instance, a child sharing three sticks of clay among four people may add to the quantity to be shared (i.e., adding a fourth stick so that each person receives one stick), create unequal shares (e.g., three of the four people receive one whole stick of clay and one of the four people does not receive a share), or not exhaust the quantity to be shared (e.g., give a half from the first two sticks to each sharer yet do not share the third stick).

[1] KUs associated with fractional quantity include (a) the notion that the whole is divisible, (b) the ability to determine the number of necessary parts before activity, (c) the notion that the parts must exhaust the whole (no second round of partitioning and no remainders), (d) the notion that the parts and partitions are related, (e) the notion of equality of the parts, (f) the notion that the whole is invariant, and (g) the notion that the parts are wholes in and of themselves and subject to further operations.