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Making use of what's given: Children's detailing in mathematical argumentative writing

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ABSTRACT

The nature of mathematical argumentative writing (MAW) is relatively understudied, particularly in the context of early childhood. While much of the literature on MAW, and mathematical argumentation in general, has focused on the manner in which procedures and warrants are sequenced, the present study focused on the grammatical resource of mathematical detailing. Mathematical detailing involves the linguistic operationalization of given information within and across the sequencing of warrants to support a mathematical claim. The present study examined the effect of varying the complexity of the given information in two similar tasks on how children used given information in their mathematical writing. Findings indicate that increasing the complexity of the givens can encourage many children to operationalize the given information, as well as provide insight into children's communicated conceptions of the mathematics at hand.

1. Introduction

Research on writing in mathematics has provided relatively little focus on "the form of the writing produced by students or to helping them to learn to write more effectively. The focus. . .has been on the mathematical learning taking place rather than on the writing itself" (Morgan, 1998, p. 2). Indeed, there is a small but growing body of literature that suggests writing in mathematics facilitates metacognition (Fried & Amit, 2003; Pugalee, 2001, 2004), and has a positive effect on students' mathematics achievement (Bell & Bell, 1985; Cross, 2009; Jurdak & Zein, 1998). However, few studies (e.g., Clarke, Waywood, & Stephens, 1993; Miller & England, 1989) have examined the nature of such writing, how to facilitate the development of more sophisticated forms, or attended to the purpose of such writing (i.e., its genre). The present study focuses on the genre of mathematical argumentative writing (MAW) with particular attention to how given information is operationalized in such writing of young children.

Much of the literature examining how students write mathematically has focused on the importance of sequencing procedures and providing justifications in distinguishing less and more sophisticated writing (e.g., Clarke et al., 1993; Jurdak & Zein, 1998; Kosko, Wilkins, & Pitts Bannister, 2009), but has not focused on how given information is utilized in MAW. However, related research focusing on mathematical proof has touched on how features of the given information in a task may affect the construction of a proof or argument (Herbst & Brach, 2006; Knuth, Choppin, Slaughter, & Sutherland, 2002; Lin, 2005; Stylianides, 2007; Weber, 2001), but has not examined in depth how given information is operationalized throughout a written proof or argument. Examining young children's MAW, Kosko and Zimmerman (2015, submitted for publication) found that when students went beyond simply referencing the given information to continually referencing and building off

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of the given information in their description of procedures, students were more likely to demonstrate more sophisticated forms of MAW. Describing this operationalization of given information as *detailing*, Kosko and Zimmerman (2015, submitted for publication) suggest this feature as a potentially key resource for students to develop more sophisticated mathematical arguments. Given the posited importance of detailing in MAW it is the primary purpose of this study to examine the manner in which children use given information in their writing. To facilitate this purpose, children's MAW on two tasks with very similar features, but distinctly different given information were examined.

2. Writing in mathematics

Mathematical detailing is a linguistic action in which an individual refers to given information for a mathematical context or task, and integrates such information to support a claim. As such, this grammatical resource applies to a particular form of writing in mathematics. To aid the reader in understanding how the present paper is positioned regarding writing in mathematics, and particularly with regards to mathematical detailing, a general overview of research in writing in mathematics is provided.

Research on students' writing in mathematics extends across many genres, or purposes for writing (Eggins, 2004). Marks and Mousley (1990) suggest that there are numerous genres in mathematics classrooms, including but not limited to the procedural genre, recounts of events, and descriptions. Not mentioned explicitly by Marks and Mousley (1990) are various other genres including, but not limited to, word problems, two-column proofs, algebraic equations, and so forth. The grain size of what counts as a genre can vary and certain genres may overlap with others. This paper focuses particularly on the genre of mathematical argument in the written medium (referred to previously and hereafter as MAW). However, the vast majority of research on writing in mathematics has focused on other genres that, while not specifically MAW, do overlap with aspects of MAW. Such research most often focuses on expository writing (i.e., Bell & Bell, 1985; Shield & Galbraith, 1998), or "writing which is intended to describe and explain mathematical ideas" (Shield & Galbraith, 1998, p. 29). By contrast, MAW is defined in the present paper as writing intended to establish an acceptable mathematical claim of truth. This goes beyond persuasive writing which seeks to convince others because MAW seeks to establish a socially accepted value for the claim (theorems, definitions, etc.). This aligns with Peirce's (1903/1998) description of argument as sign, and allows for incorporation of descriptions by other researchers of proof as a special form of or as including mathematical argument (Balacheff, 2008; Tall et al., 2012). Further, it also allows for the aforementioned genre of expository writing to overlap with argumentative writing, since explanation of mathematical ideas can often be used in support of a larger argument.

Prior study of expository writing in mathematics has predominately focused on middle, secondary or college aged students (e.g., Aspinwall & Miller, 2001; Clarke et al., 1993; Fried & Amit, 2003). Several such studies have conceptualized expository writing as a medium for learning mathematics, rather than as a process of mathematics (Morgan, 1998). Those who have examined the nature of writing in mathematics, either more generally or specific to the expository genre, have focused primarily on how procedures and descriptions are sequenced, and how such writing has attended to features of generalization and precision. Examining middle and secondary students' expository writing, Waywood and colleagues classified students' writing as engagement in a discursive task (Clarke et al., 1993; Waywood, 1992, 1994). According to Waywood (1992), engagement included: *recounting* or restating what occurred; *summarizing* or elaborating with examples; and dialogue or discussing relations to other mathematical aspects. In a later article that examined students' writing over several years, Waywood (1994) found that students moved from generally recounting their mathematics to summarizing, and from summarizing to dialogue. Clarke et al. (1993) confirmed these findings via observation of such a trend with students ranging from grade 7 to 11 in Australia. Clarke et al. also note that most students in these grades who had little exposure to writing in mathematics initially used recounting in their writing.

Shield and Galbraith (1998) provide a similar, but perhaps more specified, description of grade 8 students' expository writing in Australia. They found that students could include *kernel statements* of base content or concepts to be conveyed, *goal statements* of what students were to explain, *links to prior knowledge* or concepts learned in earlier lessons, and *practice exercises* used to help convey mathematical information in the form of *exemplars* (procedural descriptions or worked out example). The similarity between Shield and Galbraith's (1998) classifications to that of Waywood and colleagues lay in the hierarchy of detail provided in various prompts, as well as the relative frequency of writings at the lower end of such hierarchies. Rather, students in such studies often wrote very simplistic statements that included some taken-as-given fact (kernel statements), recounted procedures or provided descriptions (recounting or goal statements), and some manner of justification (summarizing, links to prior knowledge, use of exercises or examples). It should be noted that Shield and Galbraith (1998) did not organize their coding scheme into specific genres that could be ordered as less or more sophisticated, but their focus on particular features is noteworthy given prior study on expository writing in mathematics.

Seeking to expand the knowledge base of how mathematical writing develops, Kosko and Zimmerman (2015, submitted for publication) examined the MAW of children in grades K-3. Utilizing Toulmin's argumentation scheme (Toulmin, 2003), coupled with Systemic Functional Linguistics (SFL: Halliday & Matthiessen, 2004), several classifications for MAW were identified that appeared to be hierarchical within given mathematical tasks. Following Krummheuer's approach (1995, 2007), Kosko and Zimmerman (2015, submitted for publication) focused on four of the six key elements of Toulmin's (2003) argumentation scheme (see Fig. 1). The *claim* is the proposition the arguer is seeking to establish accepted truth toward. The *grounds/data* represent the given information presented in an argument to establish a claim. *Warrants* are propositions provided as explanations or justifications of the claim, and also serve to connect the grounds to the claim. Warrants can be