Journal of Mathematical Behavior 41 (2016) 87–103

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Second graders articulating ideas about linear functional relationships

María C. Cañadas[a,*], Bárbara M. Brizuela[b], Maria Blanton[c]

[a] *Universidad de Granada, Departamento Didáctica de la Matemática, Facultad de Ciencias de la Educación, Campus de la Cartuja, s/n, 18071 Granada, Spain*
[b] *Tufts University, Department of Education, 12 Upper Campus Rd., Paige Hall, Medford, MA 02155, United States*
[c] *TERC, 2067 Massachusetts Ave, Cambridge, MA 02140, United States*

ARTICLE INFO

Article history:
Received 24 June 2014
Received in revised form 29 August 2015
Accepted 21 October 2015
Available online 21 December 2015

Keywords:
Quantities
Functional thinking
Early algebra
Elementary students

ABSTRACT

In this paper, we explore the ideas that second grade students articulate about functional relationships. We adopt a function-based approach to introduce elementary school children to algebraic content. We present results from a design-based research study carried out with 21 second-grade students (approximately 7 years of age). We focus on a lesson from our classroom teaching experiment in which the students were working on a problem that involved a linear functional relationship ($y = 2x$). From the analysis of students' written work and classroom video, we illustrate two different approaches that students adopt to express the relationship between two quantities. Students show fluency recontextualizing the problem posed, moving between extra-mathematical and intra-mathematical contexts.

1. Introduction

We now have a strong body of research regarding what early algebra entails and how elementary students understand algebraic concepts. Recent research challenges so-called limitations in children's capacities to work with algebraic topics (Brizuela & Martinez, 2012; Cai & Knuth, 2011; Molina & Ambrose, 2008). As summarized elsewhere (e.g., Blanton, Stephens, Knuth, Gardiner, Isler, & Kim, 2015b), these capacities concern children's ability to develop relational thinking; use different representations to express generalizations; represent verbal statements as algebraic equations; and reason about and represent relationships between abstract quantities of physical measures.

Some of the difficulties with algebra found at the secondary level concern students' understanding of the relationship between two data sets (MacGregor & Stacey, 1995; Warren, 2000). This understanding is key from a functions based approach, which we adopt in this paper. As Schliemann, Carraher, and Brizuela (2012) observed, "a functions-based approach to early algebra relies on the importance accorded to sets of values and to ordered pairs from a domain and target" (p. 110).

In this paper, we explore how young elementary school children in second grade (around 7 years of age) understand and represent functional relationships. We present a series of classroom episodes in which children are working with a function of the type $y = 2x$. Traditionally, the formal study of functions in algebra begins in secondary school (usually around seventh grade). However, in the United States (US), for example, algebra has more recently been recast as a longitudinal strand of thinking starting from the earliest grades of schooling (e.g., Common Core State Standards Initiative [CCSSI], 2010; National

* Corresponding author.
E-mail addresses: mconsu@ugr.es (M.C. Cañadas), Barbara.Brizuela@tufts.edu (B.M. Brizuela), maria_blanton@terc.edu (M. Blanton).

http://dx.doi.org/10.1016/j.jmathb.2015.10.004
0732-3123/

Council of Teachers of Mathematics [NCTM], 2000, 2006; National Mathematics Advisory Panel, 2008). As a result, the study of functions—an important entry point into (early) algebraic thinking (Carraher & Schliemann, 2007)—has been advocated in US learning standards for the elementary grades by, for example, the NCTM and the Common Core Mathematical Practices (CCSSI, 2010; NCTM, 2000).

2. Theoretical framework

2.1. Early algebra research

Different studies and researchers consider different ways of organizing and categorizing algebra (e.g., Bednarz, Kieran, & Lee, 1996; Blanton, Levi, Crites, & Dougherty, 2011; Drijvers & Hendrikus, 2003; Drijvers, Goddijn, & Kindt, 2011; Kieran, 1996; Kieran, 2004; Mason, Graham, Pimm, & Gowar, 1985; Usiskin, 1999). Drijvers and Hendrikus (2003) highlight that the different approaches cannot be considered completely separate because algebraic activities usually involve two or more approaches, while Bednarz et al. (1996) support the importance of a balance between the different approaches. In our work, we focus on *functional thinking* (Blanton & Kaput, 2011), which according to Warren and Cooper (2005) is one of the major components of algebraic thinking.

The results from early algebra research have paved the way for algebraic content and practices to be introduced in curricular documents for both the early (i.e., Kindergarten) and upper (i.e., grades 3–5) elementary grades in different countries in the last decades. Similar to the US, countries like Australia (Australian Curriculum, Assessment and Reporting Authority, 2011), Canada (Ontario Ministry of Education & Training, 2005), Korea (Beberly, 2004, cited by Ali & Alsayed, 2010), Japan (Watanabe, 2008), and Portugal (Canavaro, 2009; Pimentel, 2010) have made curricular recommendations regarding early algebra, endorsing an increased attention to algebraic thinking during grades K-5.

2.2. Functional thinking as a strand of early algebra research

Functional thinking might be characterized as the process of building, describing, and reasoning with and about functions (Blanton et al., 2011) and is constituted by topics, procedures, and relationships concerning functions (Rico, 2006). While functional thinking includes generalizing functional relationships between quantities[1] and representing and reasoning with those relationships to understand function behavior (e.g., Blanton et al., 2011), it also concerns the ideas of qualitative change, quantitative change, relationships between these changes, and using these relationships to solve problems (Warren & Cooper, 2005).

Functions constitute a powerful concept in mathematics because they give rise to relations and transformations of mathematical concepts (Warren & Cooper, 2005) and they are present in every aspect of quantitative science (Warren, Miller, & Cooper, 2013). Throughout the history of mathematics, functions have served a unifying role in the mathematics curriculum (Freudenthal, 1983; Hamley, 1934; Schwartz, 1990). This has also been true in other disciplines because different contents have been introduced or tackled through functions (Seldon & Seldon, 1992). Framed by a constructivist theory of learning, in which knowledge is gradually constructed on prior understandings, the approach we follow in our research is that functions constitute a way to introduce students into algebra and should be dealt with longitudinally, beginning in the *early* elementary grades (Carraher & Schliemann, 2007).

2.3. Functions for elementary grades

Even though there exist different definitions of function (e.g., Ensley & Crawley, 2006; Johnsonbaugh, 2005; Pisot & Zamansky, 1996), there is general agreement that its elements include something that varies under certain conditions (Freudenthal, 1983) and all of these definitions include variable as a key element. Variables express a quantity that can have different values in a specific numerical set. We use here variables only to refer to unknown, covarying quantities. Functions can have more than two variables; here, we focus on a function that involves two variables. In these functions, the values of one variable (dependent variable) depend on the values of another variable (independent variable). Because of the age of students in our study, we specifically focus on a linear two-variable function that can be symbolically expressed as $f(x) = 2x$,[2] and x (the independent variable) can have values in the set of natural numbers. As a consequence, the independent variable can have values in the even number set.

In this work, we focus on correspondence relationships. In a correspondence approach, the focus is on the relation between two sets and on explicitly stating a (algebraic) rule (Confrey & Smith, 1995). In contrast, in a co-variational approach the links between domain and range are spatial and relational, and the (algebraic) rule is only "a derived characteristic" (p. 79). A co-variational approach to linear functions begins with the constant first difference. Our work focuses on a correspondence

[1] In this paper we focus on functional relationships between quantities as expressed through correspondence relationships (Confrey & Smith, 1995). This work is distinct from that in the early algebra area of quantitative reasoning such as that of Dougherty (2008) and Ellis (2007) (see Carraher & Schliemann, 2007).

[2] We are not expecting second graders to represent the functional relationship symbolically in this way.