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Exploring a structure for mathematics lessons that initiate learning by activating cognition on challenging tasks☆

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ABSTRACT

While there is widespread agreement on the importance of incorporating problem solving and reasoning into mathematics classrooms, there is limited specific advice on how this can best happen. This is a report of an aspect of a project that is examining the opportunities and constraints in initiating learning by posing challenging mathematics tasks intended to prompt problem solving and reasoning to students, not only to activate their thinking but also to develop an orientation to persistence. Data were sought from teachers and students in middle primary classes (students aged 8–10 years) via online surveys. One lesson focusing on the concept of equivalence is described in detail although mention is made of other lessons. The research questions focused on the teachers' reactions to the lesson structure and the specifics of the implementation in a particular school. The results indicate that student learning is facilitated by the particular lesson structure. This article reports on the implementation of this lesson structure and also on the finding that students' responses to the lessons can be used to inform subsequent learning experiences.

1. Introduction

Teachers commonly report experiencing difficulties in incorporating problem solving and reasoning into their mathematics classrooms (Stacey, 2010) while at the same time catering for students with a wide range of prior experiences (Australian Curriculum and Assessment Reporting Authority, 2010). Rather than the common approach of starting learning sequences with simple tasks intending to move to more challenging tasks subsequently (see Tzur, 2008), we are exploring an approach based on initiating learning through a challenging task—described as activating cognition. In particular, we describe the implementation of a particular lesson structure designed to initiate learning through an appropriate challenge, effectively differentiating that challenge for particular student needs, and consolidating the learning through task variations.

The data reported below are from one aspect of a larger project[1] that is exploring the proposition that students learn mathematics best when they engage in building connections between mathematical ideas for themselves (prior to instruction

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from the teacher) at the start of a sequence of learning rather than at the end. The larger project is studying the type of tasks that can be used to prompt this learning and ways that those tasks can be optimally used, one aspect of which is communicating to students that this type of learning requires persistence on their part. Essentially the notion is for teachers to present tasks that the students do not yet know how to answer and to support them in coming to find a solution for themselves.

There are many scholars who have argued that the choice of task is fundamental to opportunities for student problem solving and reasoning. Anthony and Walshaw (2009), for example, in a research synthesis, concluded that "in the mathematics classroom, it is through tasks, more than in any other way, that opportunities to learn are made available to the students" (p. 96). Similar comments have been made by Ruthven, Laborde, Leach, and Tiberghien (2009) and Sullivan, Clarke, and Clarke (2013).

There are also scholars who have proposed that those tasks should be appropriately challenging. Christiansen and Walther (1986), for example, argued that non-routine tasks, because they build connections between different aspects of learning, provide optimal conditions for thinking in which new knowledge is constructed and earlier knowledge is activated. Similarly, Kilpatrick, Swafford, and Findell (2001) suggested that teachers who seek to engage students in developing adaptive reasoning and strategic competence, or problem solving, should provide them with tasks that are designed to foster those actions. Such tasks clearly need to be challenging and the solutions needs to be developed by the learners. This notion of appropriate challenge also aligns with the *Zone of Proximal Development* (ZPD) (Vygotsky, 1978). Similarly, the National Council of Teachers of Mathematics (NCTM) (2014) noted:

> Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature. (p. 17)

This approach was described in *PISA in Focus* (Organisation for Economic Co-operation and Development (OECD) (2014) as follows:

> Teachers' use of cognitive-activation strategies, such as giving students problems that require them to think for an extended time, presenting problems for which there is no immediately obvious way of arriving at a solution, and helping students to learn from their mistakes, is associated with students' drive. (p. 1)

The OECD (2014) explicitly connected student drive, which we associate with persistence, with higher achievement.

There are many research findings that elaborate how such advice can be implemented in classrooms, some of which is reviewed below. This report seeks to extend this advice in three significant ways: first, by investigating a specific lesson structure and particular tasks; second, by suggesting how such tasks can be adapted to accommodate differences in students' prior experiences; and third, by considering how robust learning from the challenging tasks can be consolidated.

2. The connection between the research framework and the structuring of lessons

The data reported below are informed by a framework as shown in Fig. 1, adapted from Clark and Peterson (1986), that proposes that teachers' intentions to act are informed by their knowledge, their disposition, and the constraints they anticipate experiencing. The particular focus in this article is the ways that each of these factors connect to the structuring of lessons.

One node of this framework presents decisions on lesson structure as being informed by the knowledge of the teacher. The different aspects of such knowledge, specifically teachers' knowledge of mathematics, of pedagogy and of students, are represented schematically by Hill, Ball, and Schilling (2008).

The inference is that it is more likely that teachers will intend to use challenging tasks if they understand the mathematics and its potential, are aware of approaches to implementing the tasks in classrooms and can anticipate student responses.

Another node in the framework suggests that teachers' planning intentions are informed by their dispositions including their beliefs about how students learn (Zan, Brown, Evans, & Hannula, 2006), the ways that challenge can activate cognition (Middleton, 1995), and perspectives on self goals, a growth mindset and the importance of student persistence (Dweck, 2000).

A third node proposes that the ways teachers plan are influenced by constraints that they anticipate they might experience. For example, teachers may be more likely to enact lessons based on challenging tasks if they do not fear negative reactions from students (see Desforges & Cockburn, 1987).

These three nodes interact with each other and together they inform teachers' planning intentions which in turn influence the classroom actions.

A similar perspective on the nature and relationship of these influences was described by Stein, Grover, and Henningsen (1996). They argued that the features of the mathematical task when set up in the classroom, as well as the cognitive demands it makes of students (in terms of the framework, the classroom action), are informed by the mathematical task (the planned intentions). These actions are, in turn, influenced by the teacher's goals (their beliefs), and their subject-matter knowledge and their knowledge of their students (the teachers' knowledge of mathematics and pedagogy). This then informs the mathematical task as experienced by students which creates the potential for their learning.