Journal of Mathematical Behavior 40 (2015) 75–87

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Ariel's cycles of problem solving: An adolescent acquires the mathematics register[☆]

Robert Sigley[a,*], Louise C. Wilkinson[b]

[a] *Rutgers University, Graduate School of Education, 10 Seminary Place, New Brunswick, NJ 08901, United States*
[b] *Syracuse University, Department of Reading and Language Arts, 200 Huntington Hall, Syracuse, NY 13244, United States*

ARTICLE INFO

Article history:
Available online 14 April 2015

Keywords:
Mathematics register
Communication
Language
Discourse
Problem solving
Oral and written

ABSTRACT

This analysis illustrates the interdependency between developing mathematical understanding and using the specialized language unique to the discipline to express those understandings employing the mathematics register. Our focus is on Ariel, a 7th grade bilingual student from an urban district, over an 18-month period. He develops knowledge of both mathematics and the language (both oral and written) required for his successful problem solving and communication of this process. Three questions are addressed: (1) How does Ariel display his mathematical understanding during problem solving? (2) How does Ariel use the mathematics register while solving the problem? (3) What systems of representation, including language (oral and written), does Ariel bring to solving the problem? The findings show that initially, Ariel displays his mathematical understanding during his attempt to solve a specific algebraic problem; in this exploration of understanding algebraic ideas, he seeks to identify patterns. We follow his mathematical and language learning from this initial conjecture, to his reformulation and generalization of a rule. Eighteen months later, after studying algebra, Ariel attends to the systems of representation, including both the oral and written expression of the mathematics register. The findings underscore the assertion that students may not be in full command—for production and comprehension—of the mathematics register until they understand the underlying mathematics. Teachers should be encouraged to create situations in which students deploy all of their diverse repertories of mathematics knowledge and skills to the instructional tasks at hand, and to refine their use of the mathematics register.

1. Introduction and overview

Students use multiple modes of expression, such as everyday, conversational language, drawings, as well as other modes such as graphical displays and models with manipulatives to convey their understanding of mathematical concepts, principles, and problem-solving processes (Halliday, 1978). Eventually, for the purposes of both instruction and

☆ The authors are listed alphabetically and contributed equally to this paper. Additionally: the authors wish to acknowledge the contributions of Professor Robin Danzak (Sacred Heart University) and Professor Elaine Silliman (University of South Florida) for their review and critical commentary on previous versions of this paper.

* Corresponding author. Tel.: +1 848 932 0776.
E-mail address: robert.sigley@gse.rutgers.edu (R. Sigley).

http://dx.doi.org/10.1016/j.jmathb.2015.03.001

assessment, students are expected to move to the more developed, precise linguistic encoding and expressions of those understandings—that is, the mathematics register (Schleppegrell, 2007).

The analysis presented in this paper illustrates the interdependency between developing mathematical understanding and using the specialized language unique to the discipline to express those understandings by employing the mathematics register. Our focus is on one student, Ariel, as he develops knowledge of both mathematics and of the language (both oral and written) required for his successful problem solving and communication of the problem-solving process (Sigley, Maher, & Wilkinson, 2013; Silliman & Wilkinson, 2015).

We analyze the problem solving of Ariel, a 7th grade bilingual student from an urban school district, over an 18-month period. Three issues are addressed by our analysis:

(1) How does Ariel display his mathematical understanding during problem solving?
(2) How does Ariel use the mathematics register while solving the problem?
(3) What systems of representation, including language (oral and written), does Ariel bring to solving the problem?

Initially, Ariel displays his mathematical understanding during his attempt to solve a specific algebraic problem; in this exploration of understanding algebraic ideas, he seeks to identify patterns. We follow his mathematical and language learning from this initial conjecture, to his reformulation and generalization of a rule. Eighteen months later, after studying algebra, Ariel attends to the systems of representation, including both the oral and written expressions of the mathematics register.

The analysis of one case, Ariel, is considered in the current policy context. Teachers are expected to engage their students in active conceptual understanding of the mathematics that the students are required to learn. Consequently, the significance of this work is broad. The interdependency between mathematical knowledge and competence in using the mathematics register is deemed to be an essential component of the new *Common Core State Standards (CCSS) and the Standards for Mathematical Practice* (National Governors Association Center for Best Practices & Council of Chief State School Officers, CCSSO & NGO, 2010). Thus, this interdependency is a driving force behind both major standardized assessments that are implemented nation-wide, via the *PARCC* (*Partnership for Assessment for Readiness for College and Careers*) and the *SBAC (Smarter Balanced Assessment Consortium)* (http://www.smarterbalanced.org/). These new assessments and educational standards arose from state-led initiatives to increase educational expectations for students from kindergarten to grade 12, so that all students would be college-and career-ready no later than the end of high school.

The nation-wide call for content standards explicitly requires students to use discipline-specific language to discuss and create mathematical understandings, to use mathematics to make sense of the mathematical problems, to offer arguments, to be able to understand others' reasoning and share their critiques, and finally, to use tools to build models of those understandings. Thus, both oral and written language is central to mathematical teaching and learning. Working collaboratively is more than traditional cooperative learning. For collaborative learning to thrive, students must accept that the solution is jointly built and requires exchange of ideas, including talking together. Further, the input of each person is important and the ideas of others must be heard and reviewed.

This paper consists of six sections. Following this overview, in the second section we present the perspective that frames the paper and references the current policy context for learning and assessing students' mathematical knowledge. The third section introduces key concepts and features, including the mathematics register. The fourth section focuses on the methods of data collection and analysis, and the fifth section provides the analysis. The final section offers a summary, interpretation, and discussion of implications for teaching, learning, and assessment particularly for students whose first language is not English and who are learning mathematics at the same time they are learning the language of instruction—the mathematics register in English.

2. Common Core State Standards and the language of mathematics

The mathematical standards emphasize "general, cross-disciplinary literacy expectations that must be met for students to be prepared to enter college and workforce training programs ready to succeed" (CCSSO & NGO, 2010, p. 4) by the end of high school. The anchor standards emphasize the *integration of communication processes* into the entire curriculum. These processes are reading, writing, speaking and listening, and *language*. However, language is not defined as the superordinate process, but rather it is narrowly defined as the acquisition of more general academic and domain-specific vocabulary knowledge and effective use of language conventions across modes of expression. These integrated processes function as the foundation for the grade-specific standards, which reflect "end-of-year expectations" in content areas such as mathematics (CCSSO & NGO, 2010, p. 4). Recent research (Herbel-Eisenmann et al., in press) revealed that often, teachers reported they did not know enough about classroom discourse to enact the *Standards for Mathematical Practice*.

To illustrate the complex language and cultural background knowledge required for assessing mathematical knowledge among school students, we consider the following sample test item taken from the *PARCC* (grades 6–8) (PARCC, 2014; see http://parcconline.org/samples/item-task-prototypes). One aspect of the mathematics register is that certain mathematical terms may take on different or more precise meanings in the mathematics register in comparison with other usage contexts. The following grade 6 sample item that draws on ratio concepts and the relationship of unit rates to ratios exemplifies how