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# Changing students' images of "mathematics as a discipline"

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ABSTRACT

The article describes and discusses a study of upper secondary students' changes in beliefs/views about (or images of) mathematics as a (scientific) discipline. A class of 23 students is followed over a one-year period, in which they were involved in the conduction of two specially designed teaching modules. Through questionnaires, individual interviews, and video recordings (in particular of one focus group of students), different types of changes in the students' beliefs/views about mathematics as a discipline were observed. As a final result of these detected changes, a small model for (or definition of) students' reflected images of mathematics as a discipline is proposed.



## Introduction

Often when beliefs are dealt with in the literature, it is with the purpose of improving mathematical thinking and learning (e.g. Leder & Fortasz, 2002; Rösken, Pepin, et al., 2011; Schoenfeld, 1985, 1992; Törner, 2002). Cognitive as well as affective beliefs are investigated in order to identify the "ingredients" which may equip students better for solving mathematical tasks, or making teachers capable of teaching differently and/or more effectively. Specific beliefs are identified as being advantageous in the learning of certain mathematical contents, the solving of related tasks, etc., and studies are then conducted on how to change these already existing beliefs into the more favorable ones. In this sense, beliefs are regarded as means – or *tools* – to achieve understanding in the individuals' constructive learning process. Only rarely is the act of providing students or teachers with certain beliefs, e.g. by changing existing ones, about mathematics or mathematics as an established and (scientifically) practiced discipline considered a *goal* in itself within mathematics education research (a few exceptions may be Ernest (1998), Furinghetti (1993), and Niss (1994), although they prefer to talk about "mathematical awareness", etc. instead of using the term "beliefs").

However, when looking into mathematics programs, curricula, syllabi, and teaching plans we often find descriptions addressing such aspects of a more goal-oriented dimension of students' beliefs, for example in relation to their mathematical world view by proposing some desirable beliefs that may assist in turning the students into critical citizens by providing them with intelligent and concerned citizenship, a democratic competence, and with *Allgemeinbildung* (general knowledge). But despite the existence of these descriptions, it is unclear how to actually develop such a dimension of "mathematics as a discipline" in the minds of students and further how to possibly assess students' possession of such a dimension.

For these reasons, it may be argued that the beliefs discussion lacks a more goal-oriented and normative dimension – a dimension about "beliefs about desirable beliefs", meta-beliefs we could call them – since, it appears that only by articulating them as such, i.e. as goals in themselves, may such a dimension be addressed properly from a research perspective and

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eventually lead us to answer the questions of students' development of such beliefs as well as the potential assessment of them (Jankvist, 2009b, 2012). The empirical study presented in this article may be seen as a first small step on the way to doing so. In the study, two specially designed teaching modules were implemented in Danish upper secondary school, and the development of 23 students' beliefs about – or images of – mathematics as a discipline were assessed continuously over a one year period. The research questions of the study concern the content specific changes in the students' beliefs; changes in the way the beliefs were held; the nature of the changes having occurred; and finally the potential attribution of changes to the teaching modules. But before we get to any further description of the actual study and outcomes, a few matters need to be explained about the background and framing of the study. More precisely, the study takes its departure point in two Danish goal-oriented descriptions of what I shall later come to refer to as students' *images of mathematics as a (scientific) discipline*.

## Educational background and framing

Since 1987 the Danish upper secondary mathematics program has included dimensions of mathematical modeling, the historical development of mathematics, and the nature of mathematics as a discipline (Jankvist, 2008b). Despite a clear description of and intention with these dimensions, they did not always find their way into the actual classroom teaching – and sometimes not even into the textbooks used. In 2006–2007 a new mathematics program was implemented in the Danish upper secondary school due to a reform of 2005, and as part of this program a strengthening of the three dimensions was made. In particular in regard to the historical dimension this is interesting, since this seems the one of the three that had received the least attention in teaching and textbooks. In the regulations for the new program it is said that "the students must obtain knowledge about the important sides of the interaction of mathematics and culture, science, and technology" (UVM, 2007, translated from Danish). Further, as one of nine *academic goals*, it says that the students must be able to "demonstrate [display] knowledge about the evolution of mathematics and its interaction with the historical, the scientific, and the cultural evolution" (UVM, 2007, translated from Danish). The regulations also provide a description of what is called the "identity" of mathematics:

> Mathematics builds upon abstraction and logical thinking and embraces a long line of methods for modeling and problem treatment. Mathematics is indispensable in many professions, in natural science and technology, in medicine and ecology, in economics and social sciences, and as a platform for political decision making. At the same time mathematics is vital in the everyday. [. . .] Mathematics has accompanied the evolution of cultures since the earliest civilizations and human beings' first considerations about number and form. Mathematics as a scientific discipline has evolved in a continual interrelationship between application and construction of theory. (UVM, 2007, translated from Danish)

So, when students are to display "knowledge about the evolution of mathematics", one must assume that it is within the frame of this "identity" that they are expected to do so. Thus, one purpose of the teaching of mathematics at the Danish upper secondary level is to shape the students' images of mathematics as a scientific discipline according to the above description of identity.

The rhetoric of the new Danish regulations to a large degree stem from the so-called Danish KOM-report, first published in 2002 (Niss & Højgaard, 2011 – KOM is a Danish abbreviation for *Competencies and Learning of Mathematics*). In addition to listing and describing a total of eight mathematical competencies (e.g. mathematical thinking; reasoning; representation; problem tackling; etc.), the report also points out three "second order" competencies, referred to as types of *overview and judgment* (OJ) directed toward mathematics as a subject area.

While mathematical (first order) competencies comprise "having knowledge of, understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role", or in other words a kind of "well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge", the three types of overview and judgment are "active insights into the nature and role of mathematics in the world" and Niss and Højgaard state that "these insights enable the person mastering them to have a *set of views* allowing him or her overview and judgement of the relations between mathematics and in conditions and chances in nature, society and culture" (Niss & Højgaard, 2011, pp. 49, 73, my emphasis). Hence, such a "set of views" would be in relation to students' beliefs in the sense that they would assist in providing students with a more balanced and multifaceted image of mathematics as a discipline. The three types of overview and judgment are:

OJ1 – the actual application of mathematics in other subject and practice areas;
OJ2 – the historical evolution of mathematics, internally as well as in a societal context;
OJ3 – the nature of mathematics as a subject [discipline].

OJ1 concerns actual applications of mathematics to extra-mathematical purposes within areas of everyday life, society, or other scientific disciplines. The focus of OJ2 is on the actual fact that mathematics has developed in culturally and socially determined environments, and is subject to the motivations and mechanisms which are responsible for this development. According to Niss and Højgaard it is obvious that if overview and judgment regarding this development is to have any weight (or solidness), it must rest on concrete examples from the history of mathematics. OJ3 concerns the fact that mathematics