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A task that elicits reasoning: A dual analysis

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ABSTRACT

This paper reports on the forms of reasoning elicited as fourth grade students in a suburban district and sixth grade students in an urban district worked on similar tasks involving reasoning with the use of Cuisenaire rods. Analysis of the two data sets shows similarities in the reasoning used by both groups of students on specific tasks, and the tendency of a particular task to elicit numerous forms of reasoning in both groups of students. Attributes of that task and ways that those attributes can be replicated in other domains may have implications in the teaching of early reasoning.

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1. Introduction

Researchers and educators internationally have advocated that a primary goal of mathematics education of grades K-12 should be the development of reasoning and mathematical proof (Ball, Hoyles, Jahnke, & Movshovitz-Hadar, 2002). Similarly, *The Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) points out that students must be exposed to different forms of reasoning and must learn to choose and use appropriate forms of reasoning, citing: "Students need to encounter and build proficiency in all these forms [e.g., reasoning by contradiction, cases, and direct deductive reasoning] with increasing sophistication as they move through the curriculum" (NCTM, 2000, p. 59).

The ability to reason is crucial for students to develop both a need and appreciation for making convincing arguments. It is also a basic requirement for supporting arguments in justification and proof-making in the learning of mathematics. However, researchers have noted that many difficulties inhibit reasoning and proof from being emphasized in the classrooms (Ball et al., 2002). For example, many mathematics curricula and classrooms emphasize the learning of algorithms and procedures in the elementary and middle grades, and children are forced to relinquish their own ideas and disconnect

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content from its underlying concepts (Kamii & Dominck, 1998). In addition, many students view proof as a procedure or as a series of steps to follow that lack meaning and they construct proof without using logical reasoning or attempting to provide justification for their ideas (Kamii & Dominck, 1998). Consequently, children enter the upper grades ill-equipped to develop their own justifications and proofs.

As a result of these inhibiting factors, mathematicians and mathematics educators have called for the implementation of "a culture of argumentation in the mathematics classroom from the primary grades up all the way through college" (Ball et al., 2002, p. 907). In this way, emphasizing the development of reasoning in the elementary and middle grades was deemed crucial for the promotion of comprehension and the ability to communicate mathematical understanding (Hanna & Jahnke, 1996; Polya, 1981; Stylanides, 2007). This study attempts to address this need by identifying tasks that encourage the use of varied forms of mathematical reasoning (as opposed to formal proof) in the elementary and middle school, which may be precursors to the forms of proof that are used in advanced mathematics. In addition, it examines the characteristics of tasks that induce the use of such reasoning, in an attempt to pinpoint ways that other tasks can be formulated that similarly encourage the use of these forms of reasoning.

2. Theoretical framework and related literature

2.1. Students' ability to reason: the reasoning continuum

As noted in the introduction, research shows that students experience great difficulty formulating proof. However, according to Yackel and Hanna (2003), all students can make and refute claims and participate in reasoning when afforded a supportive environment. Stylanides (2007) noted further that students are afforded the opportunity to reason mathematically when they are allowed to use forms of argumentation to the best of their cognitive ability. Mathematical reasoning has been described by Yackel and Hanna (2003) as both the use of induction, deduction, association, and inference to draw conclusions about quantity and structure, and a "communal activity in which learners participate as they interact with one another to solve mathematical problems" (p. 228).

Similarly, several studies have documented young students' ability to reason effectively and provide justification for their reasoning as they work collaboratively in a supportive environment.

For example, Maher and Martino (1996) documented the reasoning of one student, Stephanie, who, as a subject of a longitudinal study that documented students' mathematical thinking, built increasingly sophisticated justifications that resulted in a presentation of an elegant proof. This one case was found to be prototypical of the justifications that were built by the students throughout the longitudinal study.[1] Results from analysis of the reasoning used by the fourth grade students who are the subject of this paper are reported in Maher and Davis (1995) and Maher, Martino, and Davis (1994). In addition, analysis of the fraction learning of these students is reported in part in Steencken (2001) and Steencken and Maher (2003). In these papers, the students were found to use powerful arguments to support their reasoning about the meaning of one half and the placement of fractions on a number line. Others (e.g. Lampert, 1990; Zack, 1997) report that fifth grade students used conjectures and reasoned effectively, incorporating valid justifications into their classroom discourse. Mueller (2007) and Yankelewitz (2009) identified and classified the forms of reasoning used by elementary and middle school students as they worked on open-ended tasks, and found that students used varied forms of reasoning and co-constructed convincing justifications of their solutions.

The disparity between the body of research on the difficulties experienced when trying to formulate formal proof (as mentioned in the introduction) and that documenting the ability of students to reason indicate that the nature of reasoning is not dichotomous, and that children's reasoning moves along a continuum of sophistication and effectiveness. The research evidences that all students have the ability to provide informal justifications, and we propose that they may be able to move along this continuum if students have experience reasoning. Finding tasks that can naturally elicit this reasoning is therefore valuable in the sense that it can provide students experience with these types of reasoning, experience students will need if they are to become proficient at reasoning in varied ways. We hypothesize that such experience, over time and with increasing sophistication and maturity, may allow students to ultimately learn to correctly formulate and understand formal proof. This paper attempts to identify a task that provides students with such an experience, and examines the aspects of the task that may be replicated to allow students other such opportunities in different mathematical contexts.

2.2. Environment that support the development of reasoning

We believe that a mathematical environment in which students interact with one another, communicating their mathematical ideas, is an ideal condition for the development of mathematical reasoning. Vygotsky's (1978) socio-cultural approach suggested that a child's reasoning process develops through social interactions with others. Cobb, Wood, and Yackel (1992) built on this claim by showing the importance of discussions in a mathematical community. According to Cobb (2000), the classroom micro-culture has a major influence on the meaning students make of mathematics and their development of

[1] For a detailed account of the results of this longitudinal study, see Maher, Powell, and Uptegrove (in press).

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