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Impacting positively on students' mathematical problem solving beliefs: An instructional intervention of short duration

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ABSTRACT

Many students of all levels of education have certain beliefs about mathematical problem solving that tend to influence negatively these students' ability or willingness to engage productively with problem solving. Previous interventions that achieved a positive impact on such student beliefs tended to last over extended periods of time, thereby producing research knowledge that is not easily amenable to "scaling up." In this paper, we take a first step toward addressing the challenging but important question of whether a positive impact on four specific student problem solving beliefs, which are common and counterproductive, can be achieved with an intervention of short duration. We focus on the implementation of a 75-min intervention in the last research cycle of a 4-year, university-based design experiment to exemplify our theoretical framework and to discuss the promise of the intervention to support the intended outcomes. Directions for future research are discussed in light of these findings.

1. Introduction

1.1. Mathematical problem solving and student beliefs

Problem solving is at the heart of mathematical activity and is important for students' learning of mathematics at all levels of education (Lampert, 2001; NCTM, 2000; Schoenfeld, 1985; Törner, Schoenfeld, & Reiss, 2007). Yet, despite the widespread recognition of the importance of mathematical problem solving, the phrase "mathematical problem" and the corresponding phrase "mathematical problem solving" have been used in different ways in the literature (Törner et al., 2007). In this paper, we use *mathematical problem* to designate

> a situation that proposes a mathematical question whose solution is not immediately accessible to the solver, because he [or she] does not have an algorithm for relating the data with the unknown or a process that automatically relates the data with the conclusion (Callejo & Vila, 2009, p. 112).

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Given this, we define *mathematical problem solving* as the activity that aims to generate a solution to a mathematical problem, that is, a response to the question in the problem that relates the data with the unknown.

Problem solving is time consuming and usually involves exploration of several unproductive solution paths before discovering a path that can lead to a solution (Carlson, 1999; Mason, Burton, & Stacey, 1982/2010; Pólya, 1945; Schoenfeld, 1985). Thus, problem solving is "cognitively demanding" (e.g., Henningsen & Stein, 1997) and cannot be channeled into predetermined and mechanical forms of activity. Rather, it requires more effort and entails higher risks of failure than other kinds of mathematical activities such as solving exercises, which lend themselves to algorithmic solution methods (Doyle, 1988).

Problem solving, though, is more than a cognitively demanding activity. It is also an activity that is heavily shaped by students' affective domain (Carlson, 1999; Furinghetti & Morselli, 2009; Schoenfeld, 1983), notably students' *beliefs*, "the set of understandings about mathematics that establish the psychological context within which individuals do mathematics" (Schoenfeld, 1985, p. 5). Carlson (1999), for example, found that the following beliefs can contribute to successful problem solving:

> [that] mathematics involves a process that may include many incorrect attempts; [that] problems that involve mathematical reasoning are enjoyable; [that] individual effort is needed when confronting a difficulty; [that] students should be expected to 'sort out' information on their own; and [that] persistence will eventually result in a solution to a problem (pp. 254-255).

According to Schoenfeld (1985), "[o]ne's beliefs about mathematics can determine how one chooses to approach a problem [...] and how long and how hard one will work on it" (p. 45). Yet, this does not mean that specific beliefs will pre-determine one's problem solving behavior, as beliefs can have a context-specific nature (e.g., diSessa, Elby, & Hammer, 2002; Hoyles, 1992) and can function as part of broader structures of related beliefs referred to as *belief systems* (e.g., Callejo & Vila, 2009). There is no consensus in the literature about a definition of "belief systems" or about how beliefs are organized in those systems, with different views having been expressed about whether the organization is "psychological but not necessarily logical" (Rokeach, 1968, p. 2), "inherently sensible" (Leatham, 2006), or "quasi-logical" (Green, 1971). Detailed discussion of the complexity of belief systems is beyond the scope of this paper, as our focus here is on specific beliefs.[2]

Existing literature on students' problem solving beliefs has identified specific beliefs that are both common and counterproductive. Next we describe four such beliefs, on which we focus in this paper and which in general terms contrast with the beliefs that Carlson (1999) identified in successful problem solvers.

First, many students believe that those "who understand the subject matter can solve assigned mathematics problems in five minutes or less" (Schoenfeld, 1988, p. 151) and tend also to perceive as unsolvable problems that are in fact within their capabilities (Muis, 2004). Second, many students believe that perseverance is not necessary for effective problem solving (e.g., Muis, 2004; Schoenfeld, 1988, 1992). The two aforementioned beliefs are related and can be characterized as low "self-efficacy beliefs" (Bandura, 1977, 1997) in problem solving, whereby *self-efficacy beliefs* denote individuals' perception of their capabilities to carry out a given task. According to Bandura (1977, 1997), beliefs of personal efficacy determine whether an individual will engage with a challenge (in this case the challenge being to solve a mathematical problem), how much effort the individual will put in meeting the challenge, and how long the effort will be sustained in the face of adversity.

The third student belief we focus on is that "[t]here are always numbers in formulations of math problems" (Callejo & Vila, 2009, p. 116). Students with this belief tend to give up working on problems that do not have numbers or other clearly identifiable mathematical referents such as formulas, because they cannot see how they can make progress with solving these problems (a case study of a student with this belief can be found in Callejo & Vila, 2009). For these students, problem solving means practicing and following rules or procedures (Muis, 2004).

The final student belief we focus on is that problem solving is not satisfying or enjoyable activity (e.g., McLeod, 1994; Philippou & Christou, 1998). This belief may relate with students' difficulties, and often failure, to cope with the high cognitive demands of problem solving (McLeod, 1994), and the slow and error-prone nature of mathematical problem solving (Doyle, 1988). It can be hypothesized that, unless students experience success in problem solving, it is unlikely that they will develop feelings of satisfaction or enjoyment in relation to problem solving (Bandura, 1977, 1997). In this sense, there may be a relationship between students' self-efficacy beliefs about problem solving (including the first two beliefs that we discussed earlier) and their belief about whether problem solving can be satisfying or enjoyable activity (Schukajlow et al., 2012).

1.2. Interventions for impacting positively on students' mathematical problem solving beliefs

The documentation of several common and counterproductive student problem solving beliefs motivated the design of interventions that aimed to impact positively on those beliefs (Perrenet & Taconis, 2009; Philippou & Christou, 1998; Schram, Wilcox, Lappan, & Lanier, 1988; Swars, Smith, Smith, & Hart, 2009). These interventions, which lasted over extended periods of time (from 10 weeks in Schram et al. to several years in the other studies), yielded positive outcomes: having participated in the interventions students developed problem solving beliefs that better approximated beliefs which are consonant with successful problem solving. In addition to showing that it *is* possible to impact positively on specific student problem solving

[2] The reader can refer to Callejo and Vila (2009), Leder, Pehkonen, and Törner (2002), or Pajares (1992).