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A formative assessment of students' algebraic variable misconceptions

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ABSTRACT

Gaining an accurate understanding of variables is one challenge many students face when learning algebra. Prior research has shown that a significant number of students hold misconceptions about variables and that misconceptions impede learning. Yet, teachers do not have access to diagnostic tools that can help them determine the misconceptions about variables that their students harbor. Therefore, a formative assessment for variable misconceptions was created and administered to 437 middle- and high-school students. Analyses from the test scores were found to exhibit strong reliability, predictive validity, and construct validity in addition to important developmental trends. Both teachers and researchers can use the test to identify students who hold misconceptions about variables.

1. Introduction

Learning algebra is a "gatekeeper" to students' future educational and career success (Adelman, 2006; RAND Mathematics Study Panel, 2003; Silver, 1997; U.S. Department of Education, 1999). An increasing number of school districts have responded recently by adding algebra to their high school graduation requirements (Achieve, 2007). Given its importance, it is disquieting that learning algebra proves so challenging. Data from the National Assessment of Educational Progress (NAEP) show that algebra achievement of U.S. students is poor, with only 6.9% of 17-year-olds scoring at or above a proficient level (National Center for Educational Statistics, 2005).

One significant problem is that many students experience difficulty mastering foundational algebraic concepts, one of which is an understanding of *variables* (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005; Kuchemann, 1978; Philipp, 1992). Moreover, misconceptions (alternative conceptions) about variables are common among students (e.g., Kieran, 1992; Kuchemann, 1978; Rosnick, 1981; Stacey & Macgregor, 1997). Yet, diagnostic assessments about variable misconceptions are not available to teachers. Therefore, the primary goal of the current study is to develop an assessment with reliable and valid items that can specifically diagnose if students harbor misconceptions about variables. The secondary goals are to (1) determine how common misconceptions about variables are among middle- and high-school students and (2) explore developmental trends in the formation of misconceptions about variables.

To understand some of the typical misconceptions that students hold about variables, it is best to begin by charting correct understanding of variables. Proper understanding of symbols as variables includes a few key components. First, the variable

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must be interpreted as representing an *unknown quantity*. That is, a student must realize that a symbol represents a unit that does not have an ascertained value. Second, a student must interpret the symbol as representing a *varying quantity* (Philipp, 1992) or range of unspecified values (Kieran, 1992). This is known as the "multiple values" interpretation of literal symbols (Knuth et al., 2005). These first two proper interpretations have been studied by presenting seventh and eight graders who have been exposed to curriculum about variables with the problem: "The following question is about the expression '$2n+3$.' What does the symbol (n) stand for?" (Knuth et al., 2005). Correct responses expressed the idea that the literal symbol (1) represents an unknown value (e.g., "the symbol is a variable, it can stand for anything") and (2) could represent more than one value (e.g., "it could be 7, 59, or even 363.0285"). However, approximately 39% of seventh graders and almost 25% of eight graders gave incorrect responses (e.g., "I don't know" or "nets" or "5"). These data provide clear evidence that a sizable group of students do not correctly interpret variables.

A third component of understanding variables entails awareness that some kind of relationship exists between symbols as their value changes in a systematic manner (e.g., as b increases, r decreases) (Kuchemann, 1978). Said differently, a correct interpretation of variables entails knowing that related numbers that change together are "variables." (Philipp, 1992). The "which is larger" problem has been relied on to assess this understanding. For example, Kuchemann (1978) presented 3000 high-school students who had been taught about variables with the following problem: "Which is the larger, $2n$ or $n+2$? Explain." Only 6% of students were correct and seemingly aware of a "second order relationship," that the relation between $2n$ and $n+2$ is actually changing with n. Indeed, the difference between $2n$ and $n+2$ increases as n increases. When $n=2$, the two expressions are equal; when $n=3$, $2n>n+2$. Knuth et al. (2005) also explored this understanding, using the "which is larger" problem with middle-school students. Only about 18% of sixth graders, just over 50% of seventh graders, and 60% of eighth graders evidenced the understanding that a relationship exists between symbols because their value systematically changes.

However, the issue is not as simple as students *lacking correct knowledge* about variables. Of additional concern is that many students actually *hold erroneous concepts* about variables. Often students come to school with knowledge of concepts in the curriculum. If this knowledge is inconsistent with the concepts being taught, the knowledge is termed an alternative conceptions or misconceptions (Lucariello, 2009). Considerable research has documented that many misconceptions in mathematics and science are quite common. The current study focuses on three of the common, major misconceptions about variables that students experience (as described in the literature) and develops an instrument that detects these three misconceptions.

The first of these misconceptions was initially documented by Kuchemann (1978) during an exploration of students' interpretations of variables. Specifically, he found some students consistently *ignored variables*. For example, in the problem "Add 4 onto $n+5$", 68% of students answered correctly ($n+9$), while 20% of students gave the incorrect answer 9, suggesting they simply ignored the variable n altogether.

A second type of misconception is seen when students treat variables as *a label for an object* (McNeil et al., 2010). This was shown by Stacey and Macgregor (1997) when they presented more than 2000 middle school students the following problem: "David is 10 cm taller than Con. Con is h cm tall. What can your write for David's height?" The correct answer is $10+h$, wherein 10 is added to the number or quantity denoted by h. Yet many students treated the variable as a label associated with the name of an object (e.g., $C+10=D$). Based on other research findings, interviews with individual students, and coding of students' informal or written explanations, Stacey and Macgregor (1997) interpreted this answer to reflect 'C' as meaning 'Con's height' and D as meaning 'David's height'. Another similar erroneous concept is seen when students interpret the variable as an abbreviated word (e.g., response of D h where the abbreviation stands for the words David's height).

This misconception of construing a variable as a label for an object is reflected also in the classic error to the "Students and Professors" problem, which reads as follows: "*Write an equation, using the variables S and P to represent the following statement. 'At this university there are six times as many students as professors.' Use S for the number of students and P for the number of professors*." An erroneous understanding that S is a label for an object (students), as opposed to a variable (number of students), led 37% of a sample of students entering college to incorrectly answer the question as $6S=P$ (Rosnick, 1981). When asked to explain this answer, students stated that they believed the answer was $6S=P$ because S was a label for *students*. (The correct answer is $S=6P$ where S stands for *number of students*.) This misconception reasoning on this "student and professor" problem was prevalent also among students already in college (Clement, Lochhead, & Monk, 1981). Another example of the misconception of a variable as a label for an object/entity is seen when students, who are given the question "*In the expression $t+4$, what doest represent*?," answer with "*time*" instead of "*any number*".

Finally, a third type of misconception is when students believe a variable is a *specific unknown* (Kuchemann, 1978; Stacey & Macgregor, 1997). In this case, students do not fully understand that a variable can represent multiple values, rather they believe it can only represent one fixed value. For example, when asked how many values p represents, students assume p can only hold one value, as opposed to many values. This contradicts the correct understanding of a variable previously discussed.

Misconceptions are particularly important for teachers to know about, as misconceptions can impede learning. The process of student learning varies contingent on whether students' preinstructional knowledge of a given concept(s) accords (or not) with correct curricular concepts (concepts in the domain). When student presinstructional knowledge is correct and consistent with correct curricular/domain knowledge, student knowledge is conceived of as "anchoring conceptions." When presinstructional knowledge is incorrect and hence runs contrary to what is being taught, student knowledge is