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The role of framing in productive classroom discussions: A case for teacher learning

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ABSTRACT

In this paper, we contrast two mathematical arguments that occurred during an algebra lesson to illustrate the importance of relevant framings in the ensuing arguments. The lesson is taken from a graduate course for elementary teachers who are enrolled in a mathematics specialist program. We use constructs associated with enthnography of argumentation to characterize the framings for warrants and backings that supported the ensuing arguments. Our findings suggest that teachers fully participated in argumentations that were framed by problem situations that were familiar to them, ones that were couched in elementary, fundamental mathematical ideas, and that these types of argumentations were arguably more productive in terms of opportunities for learning.

1. Introduction

Students should have opportunities to explain and justify ideas, listen to others' explanations, ask clarifying questions when they do not understand classmates' ideas, represent their ideas using self-invented methods, engage in challenging problem solving activities, and so on (e.g., Ball & Bass, 2003; Cobb, Yackel, & Wood, 1992; Cobb, Boufi, McClain, & Whitenack, 1997; Hiebert et al., 1997; Lampert, 1990; Rasmussen & Marrongelle, 2006; Whitenack & Knipping, 2002; Yackel & Cobb, 1996). As teachers and their students participate in these types of normative practices, they must have opportunities to build new, meaningful understandings. As Weber, Maher, Powell, and Lee (2008) state, "simply having students discuss mathematical ideas does not guarantee that meaningful learning will occur" (p. 248). For instance, as students engage in discussions, they need opportunities to "reflect on and objectify their activity as they [participate]" in discussions (Cobb et al., 1997, p. 264). Of course, the classroom teacher plays an important role as one who can facilitate learning opportunities during whole-class discussions. She takes great care in choosing appropriate representations, using students' ideas as starting points for advancing discussions, or capitalizing on ideas that underpin her students' strategies and methods (e.g., Ball, 1993; Cobb et al., 1997; Lampert, 1990). Lampert's work, for instance, highlights the challenges teachers can encounter as they teach for understanding. As she and other researchers have argued, teachers must be sensitive to students' incomplete ideas and the strategies and methods they use. For instance when students in Lampert's fifth grade class developed conjectures about the last digits in 7^4 and 7^5, she facilitated the whole class discussion as students grappled with and revised a conjecture about a numerical pattern for powers of 7. As she did so, she also made decisions about how to use students' ideas to explore more formal ideas or practices that fit with the mathematical practices in the broader community, in her case, establishing one of

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the laws of exponents (Ball, 1993; Ball & Bass 2003; Cavey, Whitenack & Lovin, 2007; Cobb & Yackel, 1996; Lampert; Yackel, 2002).

Researchers have used different sociological lenses to understand the mathematical practices that provide learning opportunities for the classroom participants (e.g., Cobb & Yackel, 1996; Lampert, 1990; Pedemonte, 2007; Stephan & Rasmussen, 2002; Yackel & Cobb, 1996). Yackel and Cobb, for instance use interactionist perspectives to account in part for "how students develop specific beliefs and values" in the mathematics classroom and, in turn, become intellectually autonomous learners (p. 458). As another example, Lampert, too, addresses how the teacher and students play important roles in establishing what it means to know and do mathematics in her classroom. These seminal works offer insights into how the mathematics classroom can be viewed as a culture in its own right, where individuals work out their obligations and rights as full participants in this community (Bauersfeld, 1995; Voigt, 1994). Our work aligns with these research efforts. We are particularly interested in understanding why some arguments appear to be more productive than others. Our hypothesis is that, in some cases, as participants refine mathematical arguments, they create possible learning opportunities that are grounded or framed in their understanding of elementary, fundamental mathematical ideas (e.g., meaning of equations, connections among addition, subtraction and place value ideas, to name a few, cf. Ma, 2010). They have opportunities to reason about the mathematics and build new connections among more advanced ideas. Our challenge is to utilize an interpretive framework that helps us make better sense of when and how these learning opportunities arise. To this end, in this paper, we attempt to answer the following research question: *Can we examine the structure of mathematical arguments to identify opportunities for student learning?*

In our discussion we use argumentation theory to illustrate how similar arguments that occurred during one of the lessons in an algebra course for elementary teachers were markedly different and provided different opportunities for learning. In our first example, Argument 1, the participants established the argumentative supports in the form of backings through an inductive process. As they engaged in this discussion, teachers had opportunities to reason quite sensibly about the behavior of the algebraic expression, *r/s*, in terms of partitive division. By way of contrast, in our second example, Argument 2, the lead instructor and several of the teachers established argumentative supports in the form of warrants for the argument by generating examples using ideas of slope. That is, the warrant, instead of the backing, was established through an inductive process. Although the participants engaged in the ensuing argument, they did not give conceptually-based explanations for the ideas that they offered. As a result, the connections that teachers made about the behavior of *r/s* as the slope of a line were somewhat limited.

One of the aims of this discussion is to contribute to the ongoing conversation about how mathematical arguments can facilitate learning in the regular university classroom (e.g., Rasmussen & Marrongelle, 2006; Stephan & Rasmussen, 2002). Like Stephan and Rasmussen, we highlight the types of argumentative supports (i.e., warrants and backings) that the classroom participants offer that advanced the argument at hand. In their work, Stephan and Rasmussen illustrate how the absence of these supports points to *collective* shifts in the classroom participants' understandings. We take a slightly different approach here. Because our focus is on how learning opportunities might arise during a given lesson, we look closely at the role that these argumentative supports play in promoting *individual* learning opportunities. As we will illustrate in this paper, learning opportunities for the teachers were constrained and/or enabled by the types of supports they and the instructor established.

As we address these issues, we first consider the theoretical constructs associated with collective argumentation (Krummheuer, 1995, 2009). We then use these ideas to examine the structural differences of two arguments from an algebra lesson. Later, we reflect on the analytic process that we used and highlight the importance of attending to the framings used by participants from a methodological standpoint.

2. Theoretical considerations

2.1. Collective argumentation: Toulmin and ethnography of argumentation

Interpretive frameworks that rely on Toulmin's (1969) scheme for making arguments have been very helpful in understanding individual and collective mathematical activity in regular classrooms or other informal settings (e.g., Whitenack and Knipping, 2002; Hollebrands, Conner, & Smith, 2010; Inglis, Mejia-Ramos, & Simpson, 2007; Krummheuer, 1995; Pedemonte, 2007; Stephan & Rasmussen, 2002; Weber et al., 2008; Yackel, 2002). Weber et al. (2008), for instance, illustrate convincingly how middle school students in an after-school program, working in small groups, can shift their reasoning to develop more general, proof-like approaches to support their claims. As students challenged others' claims they created opportunities for other students to rethink and reformulate their ideas. As another example, Inglis et al. (2007) underscore the important role that qualifiers might play in the types of warrants mathematicians used to make substantial arguments. Krummheuer (1995), too, draws on Toulmin's scheme in his theory of ethnography of argumentation. Following Toulmin (1969), he explains that persons establish an argument through a process by which they convince themselves as well as others of some claim. However, Krummheuer does not view argumentation as a singular activity in which the speaker convinces an audience of his or her ideas. He argues that from this viewpoint an individual does not need to rely on others' contributions per se. He sees this interpretation of argumentation as restrictive and one that does not adequately describe face-to-face interactions in today's classrooms (e.g., Ball & Bass 2003; Cavey et al., 2007; Stephan & Rasmussen, 2002; Yackel & Cobb, 1996). As he states,