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Elementary and middle school students' analyses of pictorial growth patterns

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ABSTRACT

Research has suggested the importance of incorporating algebraic thinking early and throughout the K-12 mathematics curriculum. One approach to help children develop algebraic reasoning is through the examination of pictorial growth patterns, which serve as a context for exploring generalization. The purpose of this study was to compare how elementary and middle school students analyze pictorial patterns, with a focus on whether students used figural or numerical reasoning. Task-based interviews were conducted with a second grader, fifth grader, and eighth grader in which they were asked to describe, extend, and generalize two pictorial growth patterns. Using a phenomenographic approach, analyses showed younger students used figural reasoning more than older students, but all students did not exclusively use figural or numerical reasoning. The students' generalizations included informal notation, descriptive words, and formal notation. The findings suggest that pictorial growth patterns are a promising tool for young students' development of algebraic thinking.

1. Introduction

Over the past 20 years, the topic of algebra in school curricula and standards documents has become increasingly emphasized as a primary content strand across K-12 mathematics (e.g., Australian Curriculum, Assessment, and Reporting Authority, 2010; National Council of Teachers of Mathematics, 2000). Traditionally, algebra has been considered a formal course taken in late middle or early high school. As recent as the late 1980s, algebra was a course deemed abstract in nature and only appropriate for college-bound students who were developmentally ready (Garmoran, 1987). In today's global economy, algebra is considered the "gatekeeper" course to higher-level mathematics and in turn, a college education. Increasingly, algebra is becoming the focus of policymakers, researchers, and practitioners with a movement toward all students taking a formal course in algebra in eighth grade (National Mathematics Advisory Panel, 2008).

Regardless of when students take algebra, students need opportunities throughout their elementary and middle school years to develop their abilities to generalize, the core of algebraic thinking (Kaput, 1999). The recently released Common Core State Standards in Mathematics (CCSS-M) in the United States outline that elementary school students should be generalizing about our number system and about other patterns in their world (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). One example is the analysis of geometric patterns (Grade 4, CCSS-M). Geometric or pictorial growth patterns serve as a rich context for exploring generalization. The purpose of this study was to explore and compare how elementary and middle school students analyze pictorial growth patterns.

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2. Background

2.1. Curriculum and standards

The Curriculum and Evaluation Standards for School Mathematics (1989) and the Principles and Standards for School Mathematics (2000) by the National Council of Teachers of Mathematics (NCTM) in the United States called for new thoughts about algebraic thinking to be integrated into work in elementary school mathematics. The *Principles and Standards* state:

> By viewing algebra as a strand in the curriculum from pre-kindergarten on, teachers can help students build a solid foundation of understanding and experience as a preparation for more-sophisticated work in algebra in the middle grades and high school. For example, systematic experience with patterns can build up to an understanding of the idea of function, and experience with numbers and their properties lays a foundation for later work with symbols and algebraic expressions (p. 37)

As outlined, NCTM proposes integrating algebraic thinking early and throughout elementary school to promote students' abilities to make generalizations. The idea of early algebra "encompasses algebraic reasoning and algebra-related instruction among young learners – from approximately 6 to 12 years of age" (Carraher & Schliemann, 2007, p. 670). More recently, the CCSS-M further support the development of algebraic thinking in elementary school students.

2.2. Describing algebra and defining algebraic reasoning

Since there is a push to integrate algebraic reasoning across the grade levels in K-12 mathematics, describing algebra and defining algebraic reasoning are necessary. First, algebra includes generalized arithmetic, a problem-solving tool, the study of functions, and modeling (Bednarz, Kieran, & Lee, 1996). Carraher and Schliemann (2007) define algebraic reasoning as "psychological processes involved in solving problems that mathematicians can easily express using algebraic notation" (p. 670). Algebra is typically thought of as a study of symbol systems while algebraic thinking is often used to indicate the kinds of generalizing that come before or during the use of formal algebraic notation (Smith, 2003). NCTM's *Principles and Standards* (2000) explain that algebraic thinking focuses on analyzing change, generalizing relationships among quantities, and representing these mathematical relationships in various ways.

2.3. Pictorial growth patterns

Making algebra accessible for all students is important for two reasons: the competitiveness of nations in our increasingly global economy and providing access to higher-salary career opportunities for minority students (Moses & Cobb, 2002; NCTM, 2000; National Math Advisory Panel [NMAP], 2008; RAND Mathematics Study Panel, 2003). Asking students to analyze contexts and pictures offers opportunities for more students to experience success in algebraic thinking well before they are enrolled in a formal algebra course. Thus, implementing activities in elementary and middle school to promote algebraic thinking is critical. One of many approaches to help young children generalize about patterns is through the examination of pictorial growth patterns (Ferrini-Mundy, Lappan, & Phillips, 1997; Friel & Markworth, 2009; Orton, Orton, & Roper, 1999). Sometimes called geometric patterns, a pictorial growth pattern is a sequence of figures that change in a predictable fashion from one figure to the next (Billings, 2008). The goal is for students to analyze, describe, and extend the pattern and ultimately, generalize about relationships in the patterns. The patterns provide a context for students to make generalizations, and the nature of the patterns allows for a variety of approaches to making generalizations.

2.4. Theoretical framework

The overarching theoretical framework that guided the development of this study's tasks is the construct of *figural concepts*, objects that have both spatial properties and conceptual properties (Fischbein, 1993). In other words, objects in the tasks (see Appendix A) have figural cues because they have been constructed in a certain way spatially. For example, the square tiles have been arranged in a square shape in the first task, and the consecutive figures relate to each other. They also have a structural property so students can generalize about a concept embedded in the figure. For example, in the second task, the structure of the figures allows students to make connections to area of a rectangle. The theory of figural concepts provides a lens for considering the modes of reasoning students might use to approach the tasks.

Two modes of reasoning are often used when analyzing pictorial growth patterns, figural and numerical. "A numerical mode of inductive reasoning uses algebraic concepts and operations (such as finite differences), whereas a figural mode relies on relationships that could be drawn visually from a given set of particular instances" (Rivera & Becker, 2005, p. 199). Often, teachers think of a procedure of manipulating numbers to develop a formula rather than using the picture to generate the formula. Therefore, teachers tend to model numerical reasoning to their students while ignoring the characteristics of the picture pattern.

When analyzing pictorial growth patterns, children often use a "covariation analysis of the pattern" (Smith, 2003) in which they focus on the changes from one figure in the pattern to the next. They use the previous figure to build up to a new figure and identify what stays the same and what changes in the pattern. This can also be called recursive induction.