Journal of Mathematical Behavior 33 (2014) 88–105

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

How students interpret and enact inquiry-oriented defining practices in undergraduate real analysis

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ARTICLE INFO

Article history:
Available online 22 November 2013

Keywords:
Defining
Inquiry-oriented instruction
Real analysis
Theory of didactical situations
Meta-mathematical values

ABSTRACT

This study investigates the influence of inquiry-oriented real analysis instruction on students' conceptions of the situation of mathematical defining. I assess the claim that inquiry-oriented instruction helps acculturate students into advanced mathematical practice. The instruction observed was "inquiry-oriented" in the sense that they treated definitions as under construction. The professor invited students to create and assess mathematical definitions and students sometimes articulated key mathematical content before the instructor. I characterize students' conceptions of the defining situation as their (1) frames for the classroom activity, (2) perceived role in that activity, and (3) values for classroom defining. I identify four archetypal categories of students' conceptions. All participants in the study valued classroom defining because it helped them understand and recall definitions. However, students in only two categories showed strong acculturation to mathematical practice, which I measure by the students' expression of meta-mathematical values for defining or by their bearing mathematical authority.

1. Introduction

A large body of mathematics education research documents how inquiry-oriented classroom practices can foster rich mathematical learning (e.g. Cobb, Wood, Yackel, & McNeal, 1992; Rasmussen & Kwon, 2007). Learning in an inquiry-oriented environment ostensibly differs from learning in more conventional classrooms because students are simultaneously engaged with the subjects of mathematical inquiry (content) and the practices of mathematical activity (process). Enacting mathematical inquiry in the classroom is meant to acculturate students into mathematical culture (i.e. practices and values) by acting as a microcosm of that community (Cobb, 1989; Yackel & Rasmussen, 2002). Upper-level undergraduate classes in particular seek to expose students to the modern culture of mathematics, which is characterized by proving theorems using precise definitions (Alcock & Simpson, 2002; Weber, 2004). A growing body of research documents the nature and effects of inquiry-oriented instruction in such proof-oriented courses (e.g. Dawkins, 2012; Fukawa-Connelly, 2012; Larsen & Zandieh, 2008; Zandieh & Rasmussen, 2010). To measure the efficacy of the enculturation process, this study investigates how students individually interpret and enact the inquiry-oriented practices endorsed in a real analysis classroom. Defining provides a fitting representative of acculturation to mathematical practice since multiple studies indicate that students' naïve use of mathematical definitions often differs from that of mathematicians (e.g. Alcock & Simpson, 2002; Edwards & Ward, 2008).

In this study, I do not fully distinguish "interpreting" and "enacting" culture because culture is created via human interaction, meaning interpreting and acting are intrinsically intertwined. One useful way of understanding this interrelationship

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http://dx.doi.org/10.1016/j.jmathb.2013.10.002

is the notion of a *frame* for a situation (Goffman, 1974). Frames describe people's perceptions of situations that help them identify the nature and goals of human interactions, such as "ordering at a restaurant," "getting pulled over by a patrolman," or "attending a mathematics class." Frames are important because they connect people's activity to identities ("what is my role in this situation?"). While many previous studies show how inquiry-oriented instruction helps students adopt mathematical ways of acting, this study contributes by further investigating students' mathematical identities as they relate to advanced mathematical practice. I shall address the following questions:

1. In an undergraduate real analysis course where the instructor endorses inquiry-oriented practices of defining, what are students': *frames* for the shared defining activity, their perceived *role* in formal mathematical activity, and the *values* that guide their mathematical defining?
2. How do students' conceptions of the defining situation (i.e. their frames, role, and values) support (conscious and unconscious) acculturation into advanced mathematical practice?

I will answer the first question based upon classroom observations and interviews with students from two inquiry-oriented real analysis classes taught by the same instructor. These classes adopted an inquiry-oriented approach to defining in the sense that they treated definitions as "under construction," consistently discussing and negotiating these formal statements. I define a classroom as "inquiry-oriented" whenever students regularly articulate key course content (e.g. definitions, theorems, proofs) prior to its direct introduction by the instructor. This definition is integral to my investigation of conceptions of the situation because such instruction breaches the common frame for the mathematics classroom in which the teacher provides all of the relevant mathematical information to the students.

However, just because the professor invites students to engage in mathematical practice does not mean that all students interpret that activity as such. Hence, I pursue the second question to understand how engagement in mathematical practice relates to the broader goal of mathematical acculturation. I investigate students' "acculturation into advanced mathematical culture" in terms of (1) their personal sense of connection to the mathematical community (reported), (2) their use of mathematical definitions (enacted) in ways compatible with mathematicians' use thereof (Alcock & Simpson, 2002; Edwards & Ward, 2008; Zaslavsky & Shir, 2005), and (3) their tendency to bear and receive mathematical authority.

2. Students' understanding and use of mathematical definitions

The current study is couched within the instructional context of undergraduate real analysis and the broader context of what is called "advanced mathematical thinking" (Tall, 1991). Being a proof-oriented course, real analysis represents a major element of undergraduate math majors' apprenticeship into the formalizing practices of the mathematical community (i.e. defining, conjecturing, proving, axiomatizing). As a result, this instructional context is especially important for engaging students in inquiry-oriented learning in which they can enact and negotiate the conventions and values of the mathematics community. A large part of first-semester real analysis concerns precisely defining concepts such as limit of a sequence, limit of a function, continuity, etc. However, these definitions are notoriously difficult for students to understand and use (e.g. Przenioslo, 2004; Roh, 2010; Tall & Vinner, 1981). The following body of literature thus situates and informs the current study of students' apprenticeship into defining practices.

2.1. Students' use of definition

A survey of the definition literature reveals a recurrent dichotomy between two primary ways in which students reason about them: either a definition describes a preexistent category or it constitutes a set of exactly those elements satisfying its conditions. Stated another way, either the category determines the defining property or the property determines the category defined (Alcock & Simpson, 2002; Murphy & Hoffman, 2012). Several such studies (Edwards & Ward, 2008; Vinner, 1991) observe that while advanced mathematical practice treats definitions as being in the latter category (*stipulated* definitions), many students treat them like the former relying more heavily on intuitive notions or prototypes (*extracted* definitions).

This is not to say that reasoning with intuitive meanings, visual representations, or examples is foreign to formal mathematical activity (Guzman, 2002). Mason and Watson (2008) emphasized the importance of generating diverse examples within a given category for students' concept development (similar to Lakatos' (1976), notion of *concept-stretching*). The mathematical community's emphasis upon formal definitions stems from the fact that "appropriate use of the definition means that any correct deductions he makes will be valid for all members of the mathematical category" (Alcock & Simpson, 2002, p. 32). To some extent, though, mathematical activity cannot transcend reasoning about the general (an example of) in terms of or by working with the particular (this example), as has been previously noted (Eisenberg & Dreyfus, 1994; Font & Contreras, 2008; Mariotti & Fischbein, 1997). For instance, Dahlberg and Housman (1997) observed that advanced undergraduate students were more successful in basic concept development about a novel definition when they engaged in example generation rather than focusing their activity upon the general statement of the concept definition alone. Furthermore, Freudenthal (1973) and Lakatos (1976) both argue that the history of mathematics shows that many definitions, though treated as stipulated in mathematical use, were devised to capture an intuitive concept in their genesis. These studies all seem to attest that presenting ready-made definitions to students ignores the significant learning that supported their initial formulation.