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Students' units coordination activity: A cross-sectional analysis

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ABSTRACT

Students' ability to coordinate multiple levels of units constitutes a cognitive core in their mathematical development across several domains, including counting, whole number multiplication, integer addition, fractions concepts, and algebraic reasoning. Identifying a progression of students' units coordination activity would help educators leverage and support student development within and across the three stages of units coordination. This paper contributes to that goal by describing a cross-section of units coordination activity observed during pairs of clinical interviews with 47 grade 6 students. Results include a 36-rank scale of units coordinating activity with accompanying characterizations and descriptors.

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Recent studies have identified students' inability to coordinate two and three levels of units as a root cause for poor performance across several domains in middle school mathematics, including fractions concepts (Hackenberg & Tillema, 2009; Izsák, Jacobsen, de Araujo, & Orrill, 2012; Norton & Boyce, 2013; Steffe & Olive, 2010), algebraic reasoning (Hackenberg, 2013; Olive & Çağlayan, 2008), and integer addition (Ulrich, 2013). The construct developed out of Steffe's work with both whole (e.g., 1992, 1994) and fractional (e.g., 2001, 2010) number learning. In his words, "I think of units coordinating as the mental operation of distributing a composite unit across the elements of another composite unit" (1992, p. 279). For example, multiplying 7 by 5 could involve the units coordination of distributing 7 units of 1 across each unit of the 5 to produce a unit (35) that, in the learner's mind, is composed of five units of seven 1s each.

Throughout this paper, we refer to stages of units coordination to indicate progressively sophisticated ways in which students coordinate units. To illustrate these stages, consider the *Vacation Problem* (Hackenberg & Tillema, 2009): "We've been on vacation for five weeks. How many days have we been on vacation?" Table 1 shows how students at each stage typically approach this problem, depending on the number of levels of units they take as given and the number of levels they coordinate in activity. "Take as given" refers to what students can assimilate into their existing cognitive structures without the need to engage in further activity (Piaget, 1970). In contrast, "in activity" refers to structures that students can build up through some kind of imagined or physical action.

We emphasize that these stages do not describe fluency with multiplication facts per se, but rather describe the psychological structures with which students operate (Piaget, 1970). For example, in a teaching experiment with a 6th grade student named Cody (Norton & Boyce, 2013), the first author (as teacher-researcher) observed that Cody could use many multiplication facts (e.g., $8 \times 3 = 24$ and $6 \times 6 = 36$) and could compute more complicated products (e.g., $7 \times 12 = 84$) by imagining the standard algorithm. Yet, when he asked Cody to determine the number of chips in 8 cups, if each cup contained 3

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Table 1
Units coordination with whole numbers.

Stage	Students' ways of coordinating units	Students' reasoning on the vacation problem
1	Students can take one level of units as given, and may coordinate two levels of units in activity	Students keep track of how many times they run through a unit of 7. For example, they might say, "7 that's 1 week; 14 that's 2 weeks; 15, 16, 17, 18, 19, 20, 21, that's 3 weeks;" etc.
2	Students can take two levels of units as given, and may coordinate three levels of units in activity	Students take the composite units of 7 and 5 as given, so they start with the goal of determining the value of five 7s. They might reason as follows: "Two 7s is 14, and two more 7s makes 28; then one more 7 would be. . . 35"
3	Students can take three levels of units as given, and can flexibly switch between three-levels-of-units structures	Students can take the quantity formed by seven 5s as given, even before they determine its value. They know this is the same quantity formed by five 7s, so they can determine the value by reasoning as follows: "Five 7s is the same as seven 5s and I'd rather work with fives because they're easier: Five 5s is 25, and two more 5s makes 35"

chips, despite repeated cues regarding the number of cups and the number of chips per cup, Cody seemed to have no way to respond except to count the chips by pointing to each cup and reciting his number sequence in groups of 3 ("1, 2, 3; 4, 5, 6; . . .")—a distinguishing feature of Stage 1 thinking.

Cody's example demonstrates that instead of referring to multiplication facts and skills, stages of units coordination refer to students' progress in constructing mental structures, like the one represented in Fig. 1 (here, "35" is conceived of as a unit composed of five units, each composed of seven units of 1), to help them understand and operate flexibly in novel mathematical situations. While Cody knew his multiplication tables and a multiplication algorithm, he had not constructed a sophisticated way of understanding and operating in situations involving, to the observer, multiple levels of units.

Students' construction of such structures has implications beyond whole number knowledge. As elaborated in the next section, students operating at Stages 1 or 2 think in ways that significantly constrain their ways of operating in the domains of fractions concepts, integer addition, and algebraic reasoning (Hackenberg, 2013; Steffe & Olive, 2010; Ulrich, 2013). These difficulties for students operating at Stages 1 and 2 become especially alarming when we consider the percentages of students at each stage entering middle school. Steffe (2007) has estimated that, by the end of fifth grade, 30–50% of students operate at Stage 1 (e.g., Cody), where the remainder of the population is comprised of students operating at Stages 2 or 3. Coordinating additional units, i.e., moving from one stage to the next, requires a substantial reorganization in students' ways of operating, which can take as long as two years (Steffe & Cobb, 1988; Steffe & Olive, 2010). Thus, we need a more fine-grained understanding of how students progress within and across stages so that educators can assess and foster incremental growth toward Stage 3 reasoning.

The purpose of this study is to map a hierarchy of units coordinating activity within and across developmental stages. What does student activity look like within Stages 1 and 2, and how does that activity develop toward Stage 3? Our data come from pairs of video-recorded clinical interviews with 47 sixth-grade students in a rural, high-needs school. We analyzed the first set of interviews with these 47 students to identify a cross-section of units coordinating activity. Our analysis resulted in 36 distinct rankings, which we tested by comparing it to students' units coordinating activity in the second set of interviews. Because we conducted interviews with 47 students, we are able to generate a more detailed mapping than anything the research community has produced so far.

1. Units coordination

Throughout their schooling experiences, students' abilities to build and coordinate units influence many facets of their mathematical thinking and activity. In this section, we elaborate on how students at each stage might coordinate units across various domains of mathematical activity, beginning with counting.

1.1. Units coordination and number sequences

"A number sequence can be thought of as a sequence of abstract unit items that contain records of counting" (Steffe, 1992, p. 263). The first two number sequences that children construct—the *Initial Number Sequence* (INS) and the *Tacitly Nested*

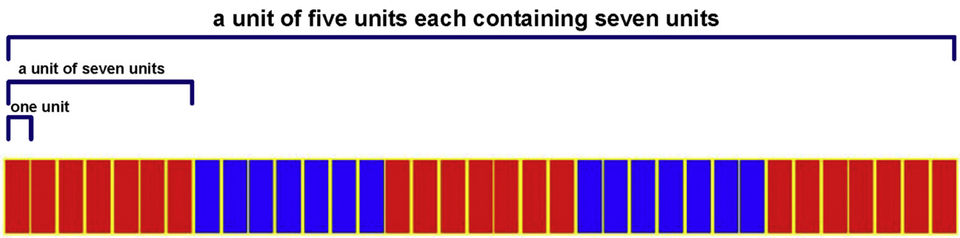

Fig. 1. A structure for coordinating three levels of units (Hackenberg & Lee, 2015).

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