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Prospective elementary teachers' claiming in responses to false generalizations

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ABSTRACT

When faced with a false generalization and a counterexample, what types of claims do prospective K-8 teachers make, and what factors influence the type and prudence of their claims relative to the data, observations, and arguments reported? This article addresses that question. Responses to refutation tasks and cognitive interviews were used to explore claiming. It was found that prospective K-8 teachers' claiming can be influenced by knowledge of argumentation; knowledge and use of the mathematical practice of exception barring; perceptions of the task; use of natural language; knowledge of, use of, and skill with the mathematics register; and abilities to technically handle data or conceptual insights. A distinction between technical handlings for developing claims and technical handlings for supporting claims was made. It was found that prudent claims can arise from arguer-developed representations that afford conceptual insights, even when searching for support for a different claim.

1. Introduction

Teachers need to be able to refute students' invalid claims to help students develop an understanding of the mathematical situation (Giannakoulias, Mastorides, Potari, & Zachariades, 2010). Studies demonstrate that students and teachers have difficulty generating appropriate refutation arguments (Balacheff, 1991; Potari, Zachariades, & Zaslavsky, 2009; Giannakoulias et al., 2010). While previous research has noted that students and teachers give problematic responses to false generalizations, the literature lacks careful attention to the claims presented and the influences on those claims.

Logically, one counterexample establishes that a generalization is false. Some literature suggests that further exploration of false statements can present opportunities for rich mathematical investigations. The basic idea is that once a counterexample is found, a student might attempt to classify all counterexamples, find counterexamples that provide insight into why the generalization is false, or develop a true generalization by altering the original claim (Peled & Zaslavsky, 1997; Komatsu, 2010; Yopp, 2013). On the other hand, as will be shown in this article, attempts to go beyond the existence of a counterexample, including making claims about classes of counterexamples and claims about cases that conform to the original claim, can lead to problematic responses when a counterexample would have sufficed. Barring two notable exceptions, Balacheff (1991) and Galbraith (1981), the literature has not addressed problematic responses developed after a counterexample has been identified.

Abbreviations: CI, conceptual insight; PST, prospective elementary (K-8) teacher; TH, technical handle.

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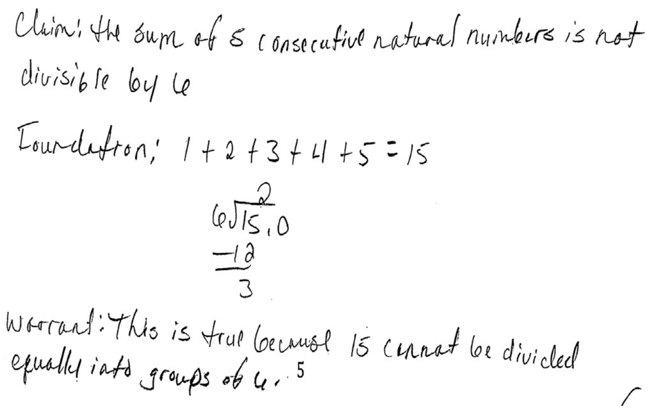

Fig. 1. Wuan's response to the prompt, "Develop a viable argument for or against a claim that the sum of five consecutive numbers is divisible by 6.".

In the United States, teachers' difficulties with communicating appropriate responses to false generalizations could prove particularly problematic for mathematics students. Common Core State Standards for Mathematics (CCSSM) call for students to construct counterexamples (NGACBP and CCSSO, 2010), and assessment designers are proposing middle-grade assessment items that ask students to test propositions or conjectures with examples (SBACS, 2013). Similar items for earlier grades may soon follow. Stylianides and Ball (2008) provide evidence that conjecture exploration and counterexample production are within the conceptual reach of children as early as third grade.

This paper explores the following research question: When faced with a false generalization and a counterexample, what types of claims do prospective K-8 teachers (PSTs) make, and what factors influence the type and prudence of their claims relative to the data, observations, and arguments reported? A "claim" is a mathematical statement that an arguer believes to be true. "Prudent claims" are claims that can be supported by the data, warrants, or conceptual insights that accompany a PST's claim. Claims are different than conjectures, in my lexicon, because conjectures have no connotation of truth. In this study I find that after PSTS acknowledge a counterexample, they often make imprudent and problematic claims. Furthermore, PSTs' ability to articulate prudent claims, with a goal of creating a viable argument, is influenced by their perception of the task; their natural language usage; their knowledge of, use of, and skill with the mathematics register; their knowledge of argumentation; their knowledge and use of the mathematical practice of exception barring; and their ability to handle data and conceptual insights appropriately and prudently.

2. The issue

In order to communicate a viable argument once they are aware of a counterexample, PSTs must report the counterexample and demonstrate that the example is indeed a counterexample, and/or they must present an alternative claim and support for that claim. This is the critical issue. How PSTs report this information can influence the correctness or appropriateness of their responses, even when the counterexample presented is otherwise correct. Student responses shown in Figs. 1 and 2 illustrate problematic reporting. These responses appeared on final exams in my mathematics courses for prospective elementary school teachers, which served as the context for this study.

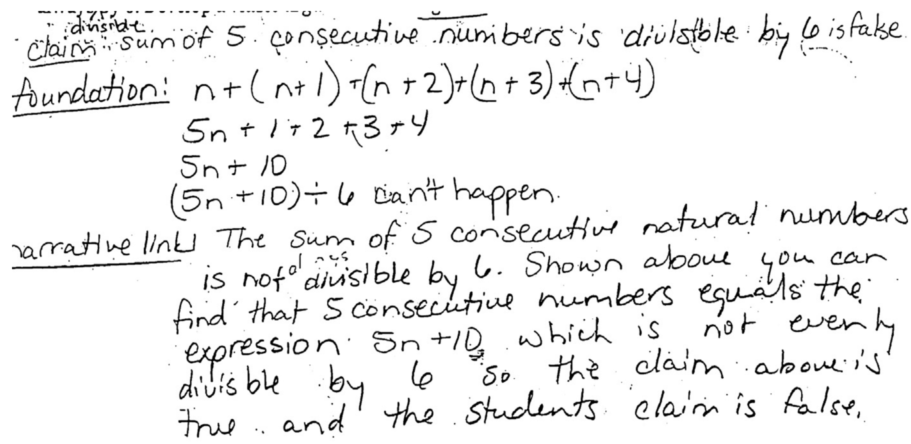

Fig. 2. Bobbi's response to the prompt, "Develop a viable argument for or against a claim that the sum of five consecutive numbers is divisible by 6.".