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Quantifying exponential growth: Three conceptual shifts in coordinating multiplicative and additive growth

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ABSTRACT

This article presents the results of a teaching experiment with middle school students who explored exponential growth by reasoning with the quantities height (y) and time (x) as they explored the growth of a plant. Three major conceptual shifts occurred during the course of the teaching experiment: (1) from repeated multiplication to initial coordination of multiplicative growth in y with additive growth in x; (2) from coordinating growth in y with growth in x to coordinated constant ratios (determining the ratio of $f(x_2)$ to $f(x_1)$ for corresponding intervals of time for $(x_2-x_1)\geq 1$), and (3) from coordinated constant ratios to within-units coordination for corresponding intervals of time for $(x_2-x_1)<1$. Each of the three shifts is explored along with a discussion of the ways in which students' mathematical activity supported movement from one stage of understanding to the next. These findings suggest that emphasizing a coordination of multiplicative and additive growth for exponentiation may support students' abilities to flexibly move between the covariation and correspondence views of function.

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1. Introduction: supporting student understanding of exponential growth

Exponential functions are an important topic both in school algebra and in higher mathematics. Not only do they play a critical role in college mathematics courses such as calculus, differential equations, and complex analysis (Weber, 2002), they also represent an important transition from middle school mathematics to the more complex ideas students encounter in high school mathematics. A focus on the conceptual underpinnings of exponential growth has increased in recent years; for instance, the Common Core State Standards in Mathematics (National Governor's Association Center for Best Practices, 2010) highlight the need to understand exponential functions in terms of one quantity changing at a constant percent rate per unit interval relative to another. Moreover, these ideas have also begun to appear at the middle school level in increasing degrees (e.g., Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006).

This paper reports on the results of a teaching experiment with middle-school students who explored exponential growth as they explored the height of a plant growing over time. The analysis reported in this paper addressed the following research questions: (a) What conceptual stages can be identified in students' exploration of exponential growth as they investigated a scenario with continuously co-varying quantities; and (b) What factors contributed to students' conceptual change over

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time? The students entered the teaching experiment with a repeated multiplication understanding of exponentiation, and by the end of the teaching sessions they were able to coordinate the ratio of y_2 to y_1 for corresponding x-values. We endeavored to identify and characterize the stages through which students progressed as they shifted from a repeated multiplication understanding of exponentiation to a covariation understanding, in which students coordinated constant ratios for y with corresponding additive changes in x. In addition, we investigated the various factors, such as problem factors and instructional moves, which encouraged particular types of mathematical activity on the part of the students. In doing so, we identified and documented three conceptual shifts in students' understanding of exponentiation that we share in this paper, followed by a discussion of the ways in which students' mathematical activity supported these shifts. The findings suggest that a covariation approach to exponentiation, in which students are encouraged to coordinate the growth factor y_2/y_1 with corresponding additive changes in x ($x_2 - x_1$), can support a flexible conception of exponential growth that includes making sense of functions such as $y = ab^x$ for non-integer exponents.

1.1. The repeated multiplication approach and challenges in understanding exponential growth

A common textbook treatment for introducing exponential growth is the repeated multiplication approach. For instance, in the middle school curriculum Connected Mathematics Project (Lappan et al., 2006), students place coins called rubas on a chessboard in a doubling pattern, and then use tables, graphs, and equations to examine the relationship between the number of squares and the number of rubas. These types of tasks require students to perform repeated multiplication to solve a problem and then to connect that process to exponential notation. This approach follows the recommendations of researchers who suggest defining exponentiation as repeated multiplication with natural numbers (e.g., Goldin & Herscovics, 1991; Weber, 2002). However, generalizing to non-natural exponents may pose difficulties for students; for instance, an expression such as $2^{2/3}$ cannot be represented using repeated multiplication, and may be difficult to understand from a repeated-multiplication perspective (Davis, 2009).

Although the literature on students' and teachers' understanding of exponential growth is limited, there is research supporting Davis' concerns about the difficulties in generalizing one's understanding of exponentiation as repeated multiplication. For instance, Weber (2002) found that college students struggled to understand or explain the rules of exponentiation and could not connect them to rules for logarithms. Pre-service teachers have not fared much better; researchers have identified their struggles not only in understanding exponential functions, but also in recognizing growth as exponential in nature (Davis, 2009; Presmeg & Nenduardu, 2005). Although pre-service teachers appear to have a strong understanding of exponentiation as repeated multiplication, they experience difficulty in connecting this understanding to the closed-form equation and in appropriately generalizing rules such as the multiplication and power properties of exponents (Davis, 2009).

Research on middle school and high school students reveals difficulties as well; students struggle to transition from linear representations to exponential representations, or to identify what makes data exponential (Alagic & Palenz, 2006). In general, exponential growth appears to be challenging to represent for both students and teachers, and it is difficult for teachers both to anticipate where students might struggle in learning about exponential properties and to develop ideas for appropriate contexts that involve exponential growth (Davis, 2009; Weber, 2002). These documented challenges suggest a need to better understand how to foster students' learning about exponential growth, and for identifying more effective modes of instruction.

1.2. Alternate approaches to exponential growth

It is possible to conceive of exponential growth in other ways besides repeated multiplication: For instance, one can approach exponential growth as products of factors (Weber, 2002) or as what Confrey and Smith (1994,1995) refer to as a multiplicative rate constructed from multiplicative units. Weber (2002) offered a theoretical analysis of exponentiation relying on Dubinsky's (1991) Action, Process, Object, Schema (APOS) theory. Although this approach initially relies on an action understanding of exponentiation as repeated multiplication, Weber describes a scenario in which students would then transition to a process understanding by interiorizing the repeated multiplication action. Students would then view exponentiation as a function and be able to reason about its properties. This would enable students to consider expressions such as 2^3 as the product of three factors of 2, and ultimately students should be able to generalize this understanding to view a^b as b factors of a. Weber's analysis offers a vision for moving beyond the repeated multiplication view of exponentiation, but it remains an open question how students might undergo these conceptual transitions.

Confrey and Smith (1994,1995) introduced an operational basis for multiplication and division called splitting, postulating a different cognitive foundation for splitting versus counting. A splitting structure is a multiplicative structure in which multiplication and division are inverse operations, such as repeated doubling and repeated halving. Within this model, students also treat the product of a splitting action as the basis for its reapplication; thus, a split can be viewed as a multiplicative unit. Confrey and Smith (1994) suggested that basing multiplication on repeated addition neglects the development of a parallel but related idea of equal sharing, magnification, or repeated copies. They assert that "[b]uilding concepts of multiplicative rates constructed from multiplicative units should play a central role as students work on understanding how multiplicative worlds generate constant doubling times and constant half-lives" (p. 55).

Splitting as an operation can form the basis of what Confrey and Smith (1994) refer to as a rate-of-change approach to exponential functions; they found a number of such approaches adopted by students making sense of exponential situations.

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