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Proof-scripts as a lens for exploring students' understanding of odd/even functions

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ABSTRACT

A group of prospective mathematics teachers was asked to imagine a conversation with a student centered on a particular proof regarding odd/even functions and produce a script of this imagined dialog. These scripts provided insights into the script-writers' mathematical knowledge as well as insights into what they perceive as potential difficulties for their students. The paper focuses on the script-writers' understandings of derivative and of even/odd functions.

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1. Introduction

There is an old adage that in order to truly understand something one has to teach it. In accord with this idea, the activity of teaching and thinking about teaching can reveal a great deal about teachers' own conceptions of the subject matter. In this study, a group of prospective secondary school mathematics teachers were invited to imagine their interaction with students about a given theorem and its proof, and describe it in a form of a script for a dialog between a teacher and a student, referred to as a *proof-script*. The presented theorem was: "The derivative of an even function is odd." Through writing a script the participants provided a glimpse into their own mathematical knowledge as well as into what they perceive as potential difficulties for their students. In particular, I focus on the script-writers' understanding of the concepts of derivative and of even/odd functions, as featured in their composed scripts.

2. Background

The research reported in this article concerns the theorem "The derivative of an even function is odd," and a particular proof of this result, which is accompanied by the diagram presented in Fig. 1. As such, brief notes on the concepts that appear in the theorem and on the use of visual representations are provided below.

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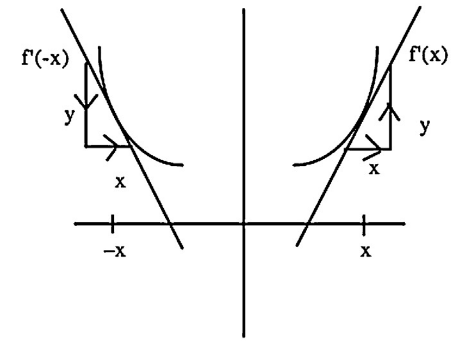
Theorem: The derivative of an even function is an odd function.

Consider the following proof of this theorem:

If f(x) is an even function it is symmetric over the y-axis.

So the slope at any point x is the opposite of the slope at (-x)

In other words, f'(-x) = - f'(x) , which means the derivative of the function is odd.



Imagine that you are working with a student and testing his/her understanding of different aspects of this proof.

What would you ask? What would s/he answer if her understanding is incomplete? How would you guide this student towards enhanced understanding? Identify several issues in this proof that may not be completely understood by a student and consider how you could address such difficulties. In your submission:

(a) Write a paragraph on what you believe could be a “problematic point” (or several points) in the understanding of the theorem/statement or its proof for a learner.
(b) Write a scripted dialogue between teacher and student that shows how the hypothetical problematic points you highlighted in part (a) could be worked out (THIS IS THE MAIN PART OF THE TASK).
(c) Add a commentary to several lines in the dialogue that you created, explaining your choices of questions and answers.

Fig. 1. The task.

2.1. On odd and even functions

There are several equivalent formulations of the definitions of odd functions and even functions. Analytically, even functions satisfy the property that $f(x) = f(-x)$. They can be defined graphically as functions that have a reflectional symmetry about the y-axis. Similarly, odd functions can be defined graphically as functions which have a 180° rotational symmetry about the origin (or have a reflectional symmetry at the point of origin), or analytically as functions for which $f(x) = -f(-x)$.

The reason these two classes of functions are given the same names as subsets of the integers is tied to exponents of monomials of the form x^n where n is an integer. When n is even $(-x)^n = x^n$, the function is symmetric about the y-axis (even function). Additionally, when n is odd $(-x)^n = -x^n$, the function has a 180° rotational symmetry about the origin (odd function). But this does not tell the whole story, since the definitions of odd and even functions are not restricted to monomials; they apply to all classes of functions that have the appropriate symmetry properties. In the case of polynomials and rational functions odd and even functions are closely related to the ideas of odd and even exponents. Any polynomial or rational function with only even exponents is an even function (e.g., $f(x) = 3x^6 - 4x^2 + x^0 - x^{-4}$) and any polynomial or rational function with only odd exponents is an odd function (e.g., $g(x) = 3x^7 + 4x - x^{-3}$). This is also related to the McLaurin series expansions of even functions, which consist of only even powers. Similarly, the McLaurin series expansions of odd functions consist of only odd powers. So the connections between odd/even functions and odd/even numbers are deeper than they may appear on the surface (Sinitsky, Leikin, & Zazkis, 2011).

Odd/even functions are important in many areas of mathematical analysis, particularly in studies of power series and Fourier series. The topic also serves as a conceptual intersection between symmetry, which is encountered most often in visual contexts such as geometry, and functions, which are often explored through analytic means. As such, odd/even functions may serve as a link between analytic and visual concepts. Bridging analytic and visual has been a goal of calculus instruction at least since the calculus reform movement of the late 1980s and early 1990s (e.g. Zimmermann, 1991).