Journal of Mathematical Behavior 35 (2014) 58–73

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Building meaning through problem solving practices: the case of four-year olds

Maria Angela Shiakalli[a], Konstantinos Zacharos[b,*]

[a] Pre-school Teacher, Cyprus
[b] Department of Educational Sciences and Early Childhood Education, University of Patras, Greece

ARTICLE INFO

Article history:
Available online 29 June 2014

Keywords:
Problem solving
Additive problems
Pre-school
Teaching intervention
Graphical representation

ABSTRACT

This paper deals with the attempt to study four-year old children's potential to engage in the mathematical problem solving process and their ability to construct mathematical meaning through their engagement with the process. We studied: (a) whether teaching could contribute to the development of the essential cognitive skills for solving a mathematical problem of detecting all possible additive combinations giving a specific sum and (b) whether children's familiarization with graphical representation practices of solutions could give meaning to the mathematical problem solving process, in a way that children would benefit from it in their involvement and effectiveness concerning the process. Findings showed that four-year old children's consistent involvement with the mathematical problem solving process enabled them to develop, apply and demonstrate skills and mathematical conceptual understanding in order to solve the given mathematical problems. The role of the teacher, in organizing the classroom learning environment as well as carrying out the teaching intervention, was essential to the development of the children's skills, abilities and mathematical conceptual understanding.

1. Introduction

Young children's school experiences are considered critical for they offer the child's first contact with the school environment within which children shape the way they perceive their role as students. This is why the interest of current school programmes focuses on the elaboration and development of effective methods for teaching and learning within the school environment (Clements & Sarama, 2009; Ginsburg & Goldbeck, 2004; Greenes, Ginsburg, & Balfanz, 2004; Shiakalli & Zacharos, 2012; Zacharos, Antonopoulos, & Ravanis, 2011; Zacharos & Kassara, 2012).

Nowadays, the development of the ability to solve mathematical problems is included amongst the basic aims of mathematical education. Solving a problem is not based on the repeated use of school practices nor on the memorization of rules and methods. More importantly, its solution process is not known forehand (Van de Walle, 2007). The children's interest to investigate an unknown, to them area, acts as an internal motive which activates them to acknowledge the agreements accompanying each problem and triggers their invention of solution strategies (Robertson, 2001).

* Corresponding author at: Department of Educational Sciences and Early Childhood Education, University of Patras, 26004 Rio Patras, Greece. Tel.: +30 2610997664; fax: +30 2610997664.
E-mail addresses: angelashiakalli@primehome.com (M.A. Shiakalli), zacharos@upatras.gr (K. Zacharos).

http://dx.doi.org/10.1016/j.jmathb.2014.05.001
0732-3123/

At the same time, problem solving functionally incorporates a variety of mathematical concepts, such as counting, graphics use, the possibility to methodically record data, the creation of positive attitudes towards working with mathematical practices (Shiakalli & Zacharos, 2012).

Current mathematics education approaches (e.g. Clements & Sarama, 2009; Simon, 2013; Simon et al., 2010; Steinbring, 2005, 2008; Voigt, 1994; Zacharos et al., 2011; Zacharos & Koustourakis, 2011) consider that the meaning given to mathematical notions chosen to be taught is the outcome of complex interactions which are developed within the micro-culture of the mathematics classroom. The forms of interaction developed within the classroom give specific meaning to mathematical practices. More specifically, reference frameworks used for teaching, the educational material and the signs and symbols mediating learning receive specific meaning within the classroom interactive practices and contribute to the giving of specific meaning in the construction of mathematical concepts (Steinbring, 2005, 2006, 2008; Zacharos & Koustourakis, 2011).

The research work presented here deals with pre-school children's involvement in the mathematical problem solving process. The presentation consists of the theoretical and conceptual framework underpinning the study, the methodology used during the study and data analysis and discussion.

1.1. The aim of the study

The aim of this study was to create a teaching intervention programme for pre-school children which would scaffold the development of cognitive skills relevant to the mathematical problem solving process. Specifically, the research questions we set to answer were:

- Can teaching contribute to the development of the essential cognitive skills for solving a mathematical problem of detecting all possible additive combinations giving a specific sum?
- Can children's familiarization with forms of graphical representations of solutions contribute to the development of effective mathematical problem solving strategies?

2. Giving meaning to mathematical practices

We have previously referred to approaches concerning the construction of mathematical concepts which underline the fact that the forms of interaction that develop within a classroom contribute towards the construction of the specific mathematical concept. Steinbring (2008) claims that mathematical concepts are constructed within a framework of interaction and they acquire their meaning within a symbolic system of structures where they are coded with the use of signs and symbols creating a logical mathematical system. Therefore teaching mathematical knowledge cannot be restricted to simple description and interpretation of mathematical communication achieved directly through the transfer of a teaching "product" which is clearly determined in advance, with specific symbolism and a strictly defined content. Steinbring (2005, 2006, 2008) claims that the mediation between mathematical signs and symbols on the one hand, and reference contexts and objects present during mathematics teaching within the classroom on the other, make up the epistemological base for the formulation of new mathematical knowledge. Thus, the epistemological interpretation of knowledge formulated during the teaching of mathematics is attributed to the relations established between symbols/signs and objects/references.

Within this framework, concerning the appropriation of mathematical knowledge, our conception about manipulatives' pedagogical role within the teaching environment alters. Their role is not simply to offer the empirical background for knowledge acquisition, but they become a part of a developmental viewing of knowledge acquisition where teaching and learning are considered a mutual process between "teacher's pedagogical knowledge and teaching activities, student's learning activities, and mathematical content influencing and thus, effectuating learning as an interactive process" (Steinbring, 2008, p. 308).

This perspective on pedagogical knowledge offered us an interpretative framework for the understanding of *how* mathematical content is constructed and *how* factors involved in educational praxis such as teachers, learners, content and symbolic representations interact during the processes of teaching and learning.

3. A conceptual approach of the learning process

Mathematics education researchers have been trying to detect children's possible learning trajectories. With some differentiation among them (Baroody, Cibulskis, Lai, & Li, 2004) they suggest relevant interpretational schemata which describe these courses (Clements, Wilson, & Sarama, 2004; Clements & Sarama, 2009; Confrey, Maloney, Nguyen, Mojica, & Myers, 2009; Simon, 1995; Wilson, Mojicab, & Confrey, 2013).

Learning trajectories are considered to contribute in the correlation of children's informal knowledge (which is based on their experiences) with cognitively elaborate and scientifically accepted views about mathematics. Teachers attempt to follow these developmental progressions and create a learning environment compatible to their students' abilities. These trajectories allow us to study key elements of the educational process and investigate questions such as, "What objectives should we establish? Where do we start? How do we know where to go next? How do we get there?" (Clements & Sarama, 2009, p. 3).