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Reading mathematics for understanding—From novice to expert

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ABSTRACT

As students progress through the college mathematics curriculum, enter graduate school and eventually become practicing mathematicians, reading mathematics textbooks and journal articles appears to become easier and leads to increased proficiency and understanding. This study was designed to begin to understand how mathematically more advanced readers read for understanding in mathematical exposition as it appears in textbooks compared to first-year undergraduate students. Three faculty members and three graduate students participated in this study and read from a first-year graduate textbook in an area of mathematics unfamiliar to each of them. The observed reading strategies of these more mathematically advanced readers are compared to observed reading strategies of first-year undergraduate students from an earlier study. The reading methods of the faculty level mathematicians were all quite similar and were markedly different from those that have been identified for undergraduate students, as well as from those used by the graduate students in this study. A Mathematics Reading Framework is proposed based on this study and previous research documenting the strategies that first-year undergraduate students use for reading exposition in their mathematics textbooks.

1. Introduction

Many would agree that reading is critical for gaining understanding within a discipline. Yet, most teachers of first-year college level mathematics courses are aware that few students do so with understanding or to the extent expected, even if readings from the textbook are assigned or required (Mesa & Griffiths, 2012; Weinberg, Wiesner, Benesh, & Boester, 2012). On their part, students complain about how hard it is to read their mathematics textbooks, and it appears that even good readers of general text are not necessarily good readers of their mathematics textbooks (Shepherd, Selden, & Selden, 2012). But as students continue in mathematics courses through their undergraduate and graduate studies, and eventually become mathematicians, somehow they learn to read mathematics textbooks and similar writings in journals with deep understanding and even enjoyment. In an effort to better understand this transition from reluctant and unskilled to engaged and proficient reading, this paper documents the strategies used by more mathematically advanced readers for learning from exposition. The result is a Mathematics Reading Framework that describes (some of the) reading strategies employed by less and more advanced readers of mathematics.

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2. Literature and theoretical perspective

We adopt the theoretical perspective that reading is an active process of meaning-making in which knowledge of language and the world are used to construct and negotiate interpretations of texts (Flood & Lapp, 1990; Palincsar & Brown, 1984; Rosenblatt, 1994). Rather than focusing exclusively on the written word from the viewpoint of the author, the emphasis is instead on how the reader interacts with the text and on the richness of the personal meanings that are constructed as part of the process (see Weinberg and Wiesner, 2010 for a detailed discussion of reader-oriented theory) Of course, the strategies that readers use as they attempt to make meaning from a text depend greatly on the individual reader, the reader's goals and the material being read (Pressley & Afflerbach, 1995). For the purposes of our work, we focus on an individual reading from a textbook with the specific goal of learning new content.

Because reading is such a major component of instruction and reading comprehension is essential to successful learning, we need to understand how academic texts are read in all disciplines, including science, technology, engineering, and mathematics. Indeed, in 2010, *Science* devoted a special issue to the challenges of reading the academic language of science. Researchers agreed that, while students have mastered the reading of various genres of English texts (mostly narratives), their reading strategies do not suffice for texts which are precise and concise, avoid redundancy, use sophisticated words and complex grammatical constructions, and have a high density of information-bearing words (Snow, 2010, p. 450). It is worth noting that many of these traits of science texts are common to mathematical exposition.

However, despite the strong similarities between mathematical and scientific text (Barton & Heidema, 2002), there has only been scant research on how students read exposition in mathematics textbooks. Osterholm (2008) surveyed 199 articles on word problems and reading in relation to problem solving but found little about reading comprehension of more general mathematical exposition. Although there is some research investigating how students read passages of mathematical exposition out of context (Osterholm, 2005, 2008), there is much less work addressing how mathematical exposition in context (e.g., in textbooks) is read and comprehended (cf. Shepherd et al., 2012).

According to both self-reported use of mathematics textbooks (Weinberg et al., 2012) and observational evidence (Randahl, 2012), students generally skip the exposition and concentrate instead on the worked examples. If reading is a goal-directed activity, identifying relevance is paramount to comprehension (McCrudden, Magliano, & Schraw, 2011, p. 2), yet students seem to see little or no instrumental value in reading mathematical exposition. Regardless of the reasons why students do not currently engage in regular reading, findings such as these may give the false impression that studying how students read mathematical exposition will not have a noticeable impact. However, instead of shying away from the issue, some researchers have worked on identifying the strategies that lead to increased comprehension of mathematical exposition with the goal of helping students improve their reading habits and ultimately their ability to learn.

Shepherd et al. (2012) identified several strategies used by undergraduate students while reading mathematical exposition. Some of these strategies were the same as those identified as being effective in a meta-analysis of reading studies that used a think-aloud protocol (Pressley & Afflerbach, 1995). Although not always performed successfully, the students attempted paraphrasing, recalling prior knowledge, and rereading phrases when confused. Other strategies not exhibited by the students, and also not identified by Pressley and Afflerbach (1995), included referencing accompanying figures or graphs, working out the examples (unless specifically directed to do so), and paying close attention to formulas and definitions. The students' failure to successfully employ effective reading strategies, together with their use of ineffective strategies, can be at least partially explained by their self-reported inexperience in the practice of reading the textbook for understanding, which is consistent with the findings of Weinberg et al. (2012) and Randahl (2012).

One of the motivations for identifying a set of routinely employed strategies for engaging in some activity is to describe the differences between novice and expert behavior. For instance, experts are better at seeing the underlying principles of a problem than novices who focus more on surface features (Chi, Feltovich, & Glaser, 1981) and employ more metacognitive strategies that keep them from continuing down a wrong solution path (Schoenfeld, 1992). Furthermore, these metacognitive strategies can be taught at the undergraduate level (Schoenfeld, 1992). The goal of this paper is therefore to identify a set of strategies used by more mathematically advanced readers ("experts") and describe some of the differences between these and the strategies used by less mathematically advanced readers ("novices") as they seek to learn from mathematical exposition. This work can then inform interventions that move students along the trajectory from novice to expert readers of their mathematics textbooks and other instructional materials.

3. Research questions

There is considerable reason to believe that most mathematicians can read mathematics textbooks and other mathematical exposition effectively. This must be done, not only to teach new courses, but to support a mathematician's own research. However few mathematicians seem to have received any instruction in reading mathematics and most seem to have tacitly learned effective reading strategies. Although we would like to eventually know why mathematicians appear to be effective readers and first-year college students are not, we limit our research questions for this study to attempting to understand some differences that mathematicians have in approaches to reading mathematics versus both first-year and advanced mathematics students and whether any observed differences seem to contribute to mathematicians' apparent ability to learn from reading mathematical exposition. Our questions are, then: