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The model method: Students' performance and its effectiveness

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ABSTRACT

This study describes Singapore students' ($N = 607$) performance on two tasks in a recently developed Mathematics Processing Instrument (MPI). The MPI comprised tasks sourced from Australia's NAPLAN and Singapore's PSLE. This study also examines students' use of the model method to solve the two tasks. The model method is a visual problem-solving heuristic prevalently used in Singapore classrooms. The study found that students who solved the tasks using a visual method predominantly used the model method as a visual problem-solving strategy. Another interesting observation was the hindrance of successful problem solving caused by the persistence of prototypical images of model drawings. Implications include encouraging teachers to get their students to identify problem situations where the model method will both work and not work well, and making the role of the generator in the model method explicit in the mathematics textbooks.

1. Background

Since the 1980s problem solving has featured prominently in most mathematics education curricula globally, with heightened interest in Singapore in recent years (Curriculum Planning and Development Division, 2006). With advances in technology, students are being exposed to a diverse range of mathematics problems in classrooms and beyond, with tasks that embed graphics (including diagrams, tables and charts) being increasingly common (Lowrie & Diezmann, 2007). Nevertheless, in some countries, and especially countries like Singapore, students are exposed predominantly to non-graphic problems (including word problems). Even without graphics, understanding the spatial information and relationships among the elements of a problem is an essential component in problem solving. In order to begin solving the problem, students are required to not only understand the relationship among elements in the problem, they also need to be able to interpret how these elements are related to each other. It is important to note that processing information in non-graphic items is high since graphics are not provided in the problem to show (or link) the relationships among elements in the problem. In other words, the ability to solve word problems requires more than having computation skills. The ability to represent the problem, often through the use of a diagram, is critical in problem solving. In addition, many studies have shown that one of the strategies used during problem solving involves using a visual method, irrespective of the type of problem – graphic or non-graphic (Krutetskii, 1976; Lean & Clements, 1981; Suwarsono, 1982). A visual method, as defined by Presmeg (1986a), is one that contains a visual image – either in the mind's eye or a diagram – which is an essential component of the solution.

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Therefore, it is important for classroom teachers to make explicit to their students the visual–spatial demands when teaching problem solving.

2. Contextual and theoretical framework

In this section, we discuss the employment of visual processing to solve mathematics tasks. We then discuss how Singapore primary school students use the model method, a "draw a diagram" heuristic commonly taught in Singapore classrooms, to solve problems.

2.1. Employing visual processing to solve mathematics task

Bruner (1964) suggested that to understand something, we either need to do it (the enactive mode); or have a picture or image of it (the iconic mode); or symbolise it through language (the symbolic mode). An individual may have a preference for a method when solving mathematics tasks. There has been an extensive body of literature (since the seminal work of Krutetskii, 1976), that suggests that non-visual (analytic) methods are the most efficient way of solving most word-based mathematics tasks. On the other hand, visual methods were found to be most appropriate (and recommended) when the need for processing is high and the problem solver is faced with complex or novel situations (Ho, 2009; Lowrie & Kay, 2001; Pirie & Kieren, 1992).

Goldin and Shteingold (2001) argued that representations can be classified as either *internal* and *external*. Internal representations are commonly classified as pictures "in the mind's eye" (Kosslyn, 1983) and include various forms of concrete and dynamic imagery (Presmeg, 1986a, 1986b) associated with personalised, and often idiosyncratic, ideas, constructs and images. External representations include conventional symbolic systems of mathematics (such as algebraic notation or number lines) or graphical representations (such as graphs and maps). These two systems cannot and do not exist as separate entities and are seen as "a two-sided process, an interaction of internalization of external representations and externalization of mental images" (Pape & Tchoshanov, 2001, p. 119).

Internal representations often involve the process of encoding information – a self-generated representation of information. Encoding techniques include drawing diagrams, visualisation and spatial reasoning. As Lowrie (2012, p. 151) maintained, "these techniques provide students with the opportunity to understand all the elements of any given problem in a way that is meaningful to them". For example, drawing a circle and dividing it into segments in order to better understand a fraction problem. By contrast, "decoding techniques are used to make sense of information within a given task, when the information has been represented visually for others to solve (e.g., interpreting a map to determine the coordinate position of a specific street crossing)" (Lowrie, 2012, p. 151).

Although encoding techniques (including visualisation and drawing a diagram) are considered supportive in scaffolding and organising information, there are some limitations in an over reliance on such techniques. For example, spatial concept acquisition and problem-solving approaches can be hindered by the persistence of "concrete" prototypical images formed in the learner's mind's eye, especially if the learner is unable to control his prototypical image during problem solving (Aspinwall, Shaw, & Presmeg, 1997: Jones, Fujita, & Kunimune, 2012; Richardson, 1999). This persistence of a prototypical image or "uncontrollable image" is one of difficulties Presmeg (1986a, 1986b) encountered with students attempting to visualise. She cautioned that such uncontrollable images may induce inflexible thinking which prevents the recognition of non-standard diagrams or non-prototypical images, thus preventing the opening up of more thoughtful avenues of thought.

The manner in which an object or a concept is represented plays an important role in the learning process. Stieff, Ryu, Dixon, and Hegarty (2012), for example, investigated students' strategy use when solving spatial problems in organic chemistry. They found that even though the tasks given were highly spatial, students employed a range of heuristics and constructed external (concrete) diagrams rather than relying on visualising/imagining to solve these tasks. The results of the study also indicated that a student's approach to solving tasks – analytic or visual-imaginistic – is independent of spatial ability, and most likely associated with the understanding (prior knowledge) and proficiency a student brings to the task, rather than having a particular preference or cognitive style (Hegarty & Kozhenikov, 1999).

2.2. The teaching of heuristics in Singapore schools

There are eight primary aims of mathematics education in Singapore schools. One of these aims is to "Develop the mathematical thinking and problem solving skills and apply these skills to formulate and solve problems" (Curriculum Planning and Development Division, 2006). To achieve the above aim, not only are primary school students taught basic mathematical concepts, processes and skills, they were also taught problem solving heuristics. The Singapore primary mathematics curriculum recommended four categories of heuristics, namely:

- To give a representation (e.g., draw a diagram, make a list, use equations),
- To make a calculated guess (e.g., guess and check, look for patterns, make suppositions),
- To go through the process (e.g., act it out, work backwards, before–after), and
- To change the problem (e.g., restate the problem, simplify the problem, solve part of the problem).