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The case of the Case of Benny: Elucidating the influence of a landmark study in mathematics education

Keith R. Leatham*, Tyler Winiecke

Department of Mathematics Education, Brigham Young University, 193A TMCB, Provo, UT 84602, United States

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ABSTRACT

Stanley Erlwanger's Case of Benny is seen by many as particularly influential in the mathematics education research community. This paper reports the results of a study designed to describe the nature of that influence. Through an analysis of academic references to the Case of Benny from the past 40 years, five primary purposes for citing the case were identified. These purposes revolve around the themes of student mathematical conceptions, the relationship between correct answers and understanding, the value of qualitative research, the impact of a behaviorist-based curriculum, and students as sense makers. The paper concludes by using these themes to reflect on the past 40 years and to look ahead to the future of research in mathematics education.

1. Introduction

It has been 40 years since the original publication of Erlwanger's (1973) "Case of Benny." In the ensuing years the work has been referred to as "classic" (Shulman, 1985, p. 442), "seminal"(Ernest, 1996, p. 805), and as "one of the most influential and important research studies in mathematics education" (Boaler, 2008, p. 592).[1] In addition, Erlwanger's Case of Benny was chosen as one of 17 articles to be included in *Classics in Mathematics Education Research* (Carpenter, Dossey, & Koehler, 2004a), a collection of articles judged to have "influenced the direction of mathematics education today" (Carpenter, Dossey, & Koehler, 2004, p. vii). This paper reports the results of a study designed to elucidate the nature of the article's influence by closely examining the ways the Case of Benny is referenced in research literature. Furthermore, the story of the influence of the Case of Benny is an interesting "case" in its own right—a compelling story that illuminates the history of research in mathematics education and the fundamental issues that have become its heart and soul.

2. Background

Stanley Erlwanger pursued his doctoral degree at the University of Illinois at Urbana, working with pioneers the likes of Robert Davis and Jack Easley. During this time he began working with Davis on his long-running NSF-funded Madison Project, the expressed purpose of which was to use "interview procedures to compare and contrast 'the mathematics in children's heads' with 'the mathematics in the school curriculum'" (Davis & Ginsburg, 1975, p. 5). In the fall of 1972 Erlwanger began

* Corresponding author. Tel.: +1 801 422 2057.
E-mail address: kleatham@mathed.byu.edu (K.R. Leatham).

[1] The Case of Benny is nowhere near the most-cited article in mathematics education literature, but it just may be one of the most read. In the author's experience, the article is routinely read by everyone who achieves an advanced degree in mathematics education.

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data collection for his dissertation, spending considerable time studying the mathematical experiences and conceptions of a 6th grade student he pseudonamed Benny. During the spring of 1973 Erlwanger conducted five additional case studies. The compilation of all six case studies constituted Erlwanger's dissertation.

Meanwhile (also in 1972), Erlwanger's dissertation advisor and mentor Bob Davis (along with Herbert Ginsburg, then at Cornell University) had begun a new journal—*The Journal of Children's Mathematical Behavior (JCMB).* In that first issue Davis and Ginsburg explained that the purpose in creating this new journal was to provide a space where open dialogue could take place around the issues of "what 'mathematical thought' means with children, how it develops, and how one might attempt to study it" (Davis & Ginsburg, 1972, p. 5). Erlwanger wrote up his original case—*Benny's Conception of Rules and Answers in IPI Mathematics*—and it was published in the second issue of *JCMB*, which appeared in the autumn of 1973 and was guest-edited by Jack Easley. This 1973 article is often referred to as the *Case of Benny*.

Erlwanger's dissertation (1974) presented the six case studies in comparison pairs. The write-up of the Case of Benny in the dissertation (paired with the Case of Mat) followed the same basic structure as the 1973 JCMB article, but was expanded by way of including more transcript excerpts and some additional analysis. In 1975 *JCMB* published the first two of those six cases (Benny and Mat) much as they appeared in the dissertation (Erlwanger, 1975), with the stated intention to subsequently publish the other four cases "because of the great interest in Erlwanger's results" (Davis & Ginsburg, 1975, p. 5). The journal never did publish these other cases, possibly because the journal itself did not publish another issue until 1977.

The Case of Benny (Erlwanger, 1973) described in great detail the mathematics-related conceptions of Benny, a 6th-grade student who "was making much better than average progress" (p. 7) in a behaviorist-based mathematics curriculum (Individually Prescribed Instruction or IPI). The IPI curriculum was based on a hierarchical sequencing of behavioral objectives. Students worked individually on exercises related to a set of objectives, asking for help when desired, then took tests to measure their competency. When students scored above 80–85% (depending on the test), they moved on to the next set of objectives; when they scored less than the threshold, they were assigned remedial exercises related to the specific items they had missed then given the opportunity to retest. As Erlwanger noted, "IPI mathematics emphasizes continuous diagnosis and assessment through pre-tests, curriculum-embedded-tests and post-tests" (p. 12).

Despite Benny's ability to attain a sufficient number of correct answers on exercises and tests related to fraction addition and decimal multiplication, Erlwanger (1973) uncovered and detailed numerous rules Benny had developed for operating on decimals and fractions that did not yield the correct answer. For example, "Benny converted fractions into decimals by finding the sum of the numerator and denominator of the fraction and then deciding on the position of the decimal point from the number obtained" (p. 8). Using this rule Benny concluded that 2/10 converts to 1.2 and that 5/10, 4/11, and 11/4 all convert to 1.5.

Beyond establishing *that* Benny was relatively successful despite his construction of erroneous rules such as this one, Erlwanger (1973) examined how such a phenomenon could exist: "How is it that Benny, with this kind of understanding of decimals and fractions, had made so much progress in IPI mathematics?" (p. 11). To answer this question he examined Benny's conception of the nature of mathematics, including his views on the nature of learning and teaching mathematics. He further explored the IPI curriculum and how it played out in Benny's classroom to illustrate how the nature of his learning environment contributed to the development of Benny's unfortunate (but seemingly sufficient for classroom success) conceptions of mathematics.

Benny's classroom success was made possible through a fascinating confluence of conceptions of mathematics and curricular design. Benny knew there were multiple equivalent representations for the fractions he was working with (he used the example of the equivalence of 1/2 and 2/4). He also knew that the answer key for his tests had a single correct answer for each problem. What Erlwanger (1973) uncovered was that Benny had combined these conceptions into "an incorrect generalization about answers" (p. 15), one that allowed him "to believe that all his answers are correct 'no matter what the key says'" (p. 15). Thus, rather than interpreting his wrong answers as wrong, he interpreted them as correct but in the wrong form. He then played a game, a "wild goose chase," (p. 16) of looking for patterns in the correct answers and "rules" that would allow him to get those answers frequently enough to get at least 80% on his mastery tests. He thus maintained numerous rules for working "different" kinds of problems, even though frequently these rules contradicted each other and resulted in numbers that actually were not equivalent. This game he played led "him to believe that the answers work like 'magic, because really they're just different answers *which we think they're different*, but really they're the same'" (p. 18).[2]

3. Methods

We began our search for references to the Case of Benny by using the Google Scholar citation reports for the original publication of the case (Erlwanger, 1973) as well as for later publications that also contain the entire case (Erlwanger, 1973/2004, 1974, 1975). We located each of these 290 publications and verified whether they truly cited Benny. Only 26 of the publications did not contain legitimate citations. We further excluded non-English-language publications as well as documents without some level of peer-review (e.g., unpublished manuscripts, class syllabi, webpages). Having relied solely on Google Scholar to create this initial collection of 221 publications, we searched for phrases like "Erlwanger Benny" and "Benny's

[2] Although these paragraphs presented a brief summary of the Case of Benny, if the reader has not already done so we recommend they read Erlwanger (1973) in order to make the current article more meaningful.