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The role of children's mathematical aesthetics: The case of tessellations

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ABSTRACT

In order to understand the role of mathematical aesthetics in the classroom and how it promotes mathematical learning, we must first understand children's current aesthetic values. Six children, ages 8–10, were interviewed as they explored tessellations for the first time. The students repeatedly used certain aesthetic themes. Further interviews revealed mathematicians invoked the same themes, though in different ways. The students' aesthetic criteria motivated their explorations, guided their creation of tessellations, and informed their evaluations. Rather than being an optional topic, mathematical aesthetics may be foundational for children's mathematical thinking, just as it is for mathematicians'.

"*If mathematical aesthetics gets any attention in the schools, it is as an epiphenomenon, an icing on the mathematical cake, rather than as the driving force which makes mathematical thinking function*" (Papert, 1980, p. 192).

1. Introduction

Mathematicians have long recognized the essential role that aesthetics plays in the doing and evaluating of mathematics (Sinclair & Pimm, 2006). Mathematics educators have echoed this importance in principle, but the role of aesthetics—not only as a motivating force for doing mathematics, but also as one of the foundations of modern mathematics—is generally overlooked in mathematics classrooms. In the last decade, some researchers in mathematics education have begun to study the importance of aesthetics for children's doing and learning of mathematics, showing that aesthetics plays a role in children's mathematical thinking as it does in mathematicians' thinking (e.g., Hersh & John-Steiner, 2011; Mack, 2006, 2007; Sinclair, 2004, 2006, 2009). This study builds on this recent research by taking a closer look at children's mathematical aesthetics for a topic they have not yet studied.

Whereas numerous researchers have attempted to analyze mathematicians' aesthetic values (e.g., Borel, 1983; Dreyfus & Eisenberg, 1986; Hardy, 1940; Krull, 1930/1987; Le Lionnais, 1948; Sawyer, 1955/1982; Schattschneider, 2006; Sinclair & Pimm, 2006; Wells, 1990), insufficient consideration has been given to the aesthetic aspects of mathematics that appeal to children. The establishment of educational goals begins with an understanding of what the child currently believes and understands about the subject. An aesthetic education therefore begins with the child's aesthetic views. Furthermore, it is often assumed (e.g., Dreyfus & Eisenberg, 1986) that the goal of any aesthetic education should be to mold the child's aesthetic to resemble the mathematician's. Researchers such as Nathalie Sinclair (2006) have challenged this idea, noting that the child's aesthetic serves its own useful purposes, similar to that of the mathematician's, but uniquely suited to the

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child's understanding of mathematics. An analysis of children's mathematical aesthetics and the role it plays in children's learning precedes realization of the role of mathematical aesthetics in education.

For this study, I investigated children's pre-instruction understandings of tessellations. This paper looks at the aesthetic perspective of their understanding—the roles children's aesthetics played in motivating and guiding them through their initial explorations of tessellations.

2. Background

2.1. Tessellations

This study is part of a larger study on children's understanding of tessellations (Eberle, 2011). An initial pilot study showed that analyzing student thinking from an aesthetic perspective could be fruitful, which is the focus of this report.

Mathematically, a tessellation (or *tiling*) is a covering of a geometric space by sets (*tiles*) which intersect only on their boundaries. For the purposes of elementary education, tessellations are usually made of polygons which cover the Euclidean plane. The polygons are all congruent to one or more *prototiles* that define the shapes that the children are to use to create the tiling. The tiles must not overlap (their interiors are disjoint) and there must be no gaps between the tiles. Most importantly, the tiling must cover an infinite plane. This is most easily achieved by finding a pattern which repeats in all directions. Such a tessellation has translation symmetry in two independent directions and is said to be *periodic*. Children also often make non-periodic radial tessellations which start with a center shape and grow symmetrically in several directions.

The artist M. C. Escher is well known for his sophisticated tessellations using tiles shaped like animals and other natural objects (Schattschneider, 2010). Works by Escher, as well as historical tilings such as those at the Alhambra in Spain, have attracted mathematicians for both their mathematical sophistication and their aesthetic appeal (Jaworski, 2006).

2.2. Mathematical aesthetics

Mathematics has been understood to be an aesthetic study since antiquity. The ancient Greeks believed that mathematics was the study of idealized perfection. For the Greeks, the very ontology of mathematics was therefore rooted in the beautiful. Today nearly all mathematicians accept that mathematics is an aesthetic study, though for a variety of reasons (Burton, 2001; Wells, 1990).

Modern mathematics is aesthetic in its foundations, its methodology, and its results (Sinclair & Pimm, 2006). It has been recognized since the modernist transformation of the late 19th century that axioms and definitions in mathematics are basically arbitrary (Gray, 2008), chosen for the primarily aesthetic reasons of simplicity and elegance, as well as for logical reasons such as consistency, independence, and relative completeness (The latter could be considered aesthetic as well).

The mathematician Henri Poincaré (1908/2000) believed that aesthetics was also the force that guided mathematicians along fruitful paths of inquiry. As the Greeks had found beauty in the ontology of mathematics, Poincaré saw that beauty was the source of its epistemology. Aesthetics is a "way of knowing" (Sinclair, 2006) mathematics which is different from the deductive reasoning we use to formally settle mathematical questions. A well-developed sense of aesthetics guides mathematicians to "see" the best paths of research long before they discover any deductive reasoning that justifies their exploration.

Few mathematicians since Poincaré have written about this generative role of aesthetics, perhaps because mathematicians can accomplish their work without introspection into the forces that direct their research. What little analysis has been done since Poincaré on mathematical aesthetics has usually focused on the static final results of mathematics (theorems and proofs) rather than the dynamic methods that led there. However, Sinclair (2004) notes that research on the importance of the affective domain in mathematics would support the importance of a more process-oriented understanding of aesthetics. Mathematicians are led by emotions such as curiosity toward what they perceive to be the more aesthetic aspects of mathematics. The ideas and methods that emotionally attract them frequently prove to be the most fruitful. Without such emotional aesthetic involvement, it might be difficult for mathematicians to persevere in extended mathematical inquiry.

Some authors have tried to analyze the criteria that mathematicians use to evaluate theorems and their proofs. Though there is no firm agreement, most mathematicians mention criteria such as significance (or depth), surprise, simplicity (clarity or elegance), connectedness, and visual appeal (e.g., Hardy, 1940; Le Lionnais, 1948; Schattschneider, 2006; Wells, 1990). We can use these criteria as an initial rubric for what mathematicians consider aesthetic even if it leaves open the question of an ultimate definition of mathematical aesthetics.

These aesthetic criteria can often be related to criteria of importance in mathematics. Simplicity makes the result easier to understand, and therefore less prone to error. Criteria such as significance and connectedness are important for identifying those elements of mathematics that will move the field forward. The criterion of surprise is perhaps related to the concept of cognitive dissonance; it implies delight in novel situations that challenge our ideas. The criterion of visual appeal is perhaps related to that of inevitability (simplicity). When we can perceive a mathematical truth with our eyes, the idea becomes more compelling. This intersection of aesthetic criteria with what is important and generative for doing mathematics suggests a close relationship between the aesthetic and the cognitive in mathematics and suggests a fundamental importance for aesthetics in genuine mathematical inquiry.