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Exploring teachers' implementation of comparison in Algebra I

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ABSTRACT

Discussions where teachers engage students in the comparison of multiple solution strategies to a single problem have been recommended in curriculum policy documents, yet integrating these discussions into teachers' normative routines is not widespread. In this paper, we begin to explore variations in teachers' implementation of Algebra I curriculum materials specifically focused on comparison. We explore (via case studies) implementation of the curriculum materials by two teachers with similar teaching backgrounds. The case studies suggest that these two teachers' implementation of the comparison materials differed markedly, raising questions about possible factors which may have contributed to implementation differences.

Mathematics curriculum policy documents in the United States argue that students should be provided with opportunities to compare and discuss multiple strategies for solving mathematics problems. For example, a recent Institute of Education Sciences practice guide focused on improving students' skills in mathematical problem solving recommends that "teachers instruct students in a variety of strategies for solving problems and provide opportunities for students to use, share, and compare the strategies" (Woodward et al., 2012, p. 32). Early research projects conducted with students in the primary grades, such as the Cognitively Guided Instruction (CGI) project, illuminated the benefits that young children can derive from comparing and discussing multiple problem solving strategies with their peers (e.g. Fennema et al., 1996). More recently, the Common Core State Standards for Mathematical Practice state that students should be able to "understand the approaches of others to solving complex problems and identify correspondences between different approaches" (National Governors Association Center for Best Practices, 2010).

However, despite policy reports and research suggesting the potential learning benefits of instruction incorporating the comparison of multiple strategies, there has been relatively little research into how the comparison of multiple solution strategies may be implemented at the introductory algebra level. Developing effective instructional strategies for Algebra I is an important policy concern, because algebra is widely considered a key "gatekeeper" subject associated with admission to advanced educational and career pathways (Moses & Cobb, 2001; Stein, Kaufman, Sherman, & Hillen, 2011).

1. Comparison of contrasting cases

One Algebra I instructional strategy which appears promising based on early studies is guiding students to compare and contrast two 'contrasting cases,' or worked examples which depict algebra problems solved using multiple strategies

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presented side-by-side. We note before proceeding that this approach differs from that explored in other research projects, such as CGI, which emphasized students' *invention* of their own strategies. The current intervention does not focus on students inventing their own strategies for Algebra I problems, but instead emphasizes students' comparison and discussion of pairs of *already-worked* examples. The current approach differs from that used in prior studies conducted at the elementary level, yet shares the goal of supporting students' conceptual and procedural learning, through participation in a structured discussion focused on specific learning goals for each comparison pair. We return to this issue below.

Results from several relatively short-duration research experiments suggest that comparing worked examples that depict multiple solution strategies side-by-side may help early algebra learners to develop their procedural flexibility, or their ability to solve problems using different strategies and to adaptively select strategies appropriate to the problem posed (Woodward et al., 2012). The National Research Council's report *Adding It Up* highlighted the importance of flexibility as a tool for problem-solving, describing flexibility broadly as "a fundamental characteristic needed throughout the problem-solving process" (Kilpatrick, Swafford, & Findell, 2001, p. 126). Procedural flexibility may be the most proximal outcome to comparison interventions (Woodward et al., 2012). For example, in two short-term randomized experiments on algebraic equation-solving, Rittle-Johnson and Star found that seventh- and eighth-grade students who were prompted to examine pairs of worked examples that presented multiple solution strategies to the same problem side-by-side, and to compare these two strategies, performed better on posttest measures of procedural flexibility than their peers who were prompted to examine the same strategies presented one after the other (Rittle-Johnson & Star, 2007), or to examine sets of similar problems solved using one strategy (Rittle-Johnson & Star, 2009). In a third study the researchers found that the effectiveness of this approach may have been sensitive to students' prior knowledge, with students who demonstrated some prior knowledge of algebra at pretest experiencing greater learning gains from comparing multiple strategies than those demonstrating no prior knowledge (Rittle-Johnson, Star, & Durkin, 2009). However, each of these interventions was relatively short, lasting only several class periods. In addition, the effects on overall conceptual and procedural outcomes varied across studies, possibly due to variability in the content presented in the short-term interventions.

One possible challenge inherent in implementing the Common Core State Standards is that the Standards call for teachers to implement instructional practices which are unfamiliar to many Algebra I teachers, such as engaging students in mathematical discussions where they compare and contrast multiple solution strategies for solving algebra problems (Silver, Ghousseini, Gosen, Charalambous, & Strawhum, 2005). Prior work has found that integrating discussions where teachers engage students in the comparison of multiple solution strategies to a single problem into teachers' normative routines is not widespread (e.g. Hiebert et al., 2003). To date, there has been relatively little exploration of how curricula emphasizing the comparison of multiple strategies (a practice recommended by the NRC and the Common Core) might be implemented in regular classroom settings with the full content of the Algebra I course.

This suggested the need for the current exploratory pilot research, to investigate how learning with comparison might be implemented outside the lab, in regular Algebra I classroom settings. In the current paper, then, we extend this work by looking at variations in middle school teachers' implementation of an Algebra I curriculum specifically focused on comparison. We explore (via case studies) two teachers whose demographic profiles and years of teaching experience were quite similar, but whose implementation of comparison was very different. Our findings raise questions about the possible roles that factors such as mathematical knowledge for teaching and beliefs about the value and goals of engaging students in comparison may have played in shaping implementation of the comparison curriculum.

In the project from which the current data are derived, we conducted an exploratory pilot study to examine teachers' use of a set of new, researcher-developed curriculum materials designed to 'infuse' the comparison of multiple solution strategies into teachers' regular, year-long Algebra I courses. As we describe in further detail below, the curriculum materials featured two cartoon characters named Alex and Morgan, who solved algebra problems using multiple strategies. As we also discuss below, the curriculum materials prompted teachers to engage students in a three-phased discussion, with question prompts in each phase designed to encourage students to (1) *understand*, (2) *compare*, and (3) *make connections* among the compared strategies.

2. Challenges to implementing reform instructional practices

Prior studies have documented myriad challenges that teachers may experience when asked to implement new instructional reforms, ranging from misalignment between teachers' attitudes and beliefs and the goals and practices envisioned in the reform (e.g. Borko, Mayfield, Marion, Flexer, & Cumbo, 1997), to limitations on resources such as curriculum materials and professional learning opportunities aligned with the new reform (e.g. Coburn, 2005; Cohen & Hill, 2000, 2001). Regarding attitudes, for example, prior research has found that even some middle grades teachers who were experienced and supportive implementers of a reform-oriented, NSF-funded mathematics curriculum expressed some doubts about the benefits of teaching with multiple strategies, and apparent reluctance to expose students to multiple strategies for unfamiliar material (Silver et al., 2005). More broadly, scholars have found that teachers' implementation of new curricula is shaped by a complex web of factors including attitudes, beliefs, prior experiences as learners and teachers, perceptions of their students, school climate, training and professional development experiences, mathematical content knowledge, and pedagogical content knowledge (e.g. Coburn, 2005; Cohen & Hill, 2001; Philipp, 2007).

Of the many factors which contribute to teachers' implementation of challenging new pedagogical practices, one factor which some scholars have highlighted as particularly influential is teachers' mathematical knowledge for teaching, or MKT.