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Teacher listening: The role of knowledge of content and students

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ABSTRACT

In this research report we consider the kinds of knowledge needed by a mathematician as she implemented an inquiry-oriented abstract algebra curriculum. Specifically, we will explore instances in which the teacher was unable to make sense of students' mathematical struggles in the moment. After describing each episode we will examine the instructor's efforts to listen to the students and the way that these efforts were supported or constrained by her mathematical knowledge for teaching. In particular, we will argue that in each case the instructor was ultimately constrained by her knowledge of how students were thinking about the mathematics.

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The challenges associated with research-based inquiry-oriented mathematics instruction have been well documented in the literature (Ball, 1993; Cohen, 1990; Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007). At the same time, there is a growing body of research on the types of specialized knowledge required for teaching (Ball, Thames, & Phelps, 2008; Hill et al., 2008; Shulman, 1986). However, much less is known about how these types of knowledge support teachers as they engage in inquiry-oriented instruction (Speer & Wagner, 2009).

One notable exception that begins to address this gap in the research base is a study done by Speer and Wagner (2009) in which they reported on the difficulties of providing *analytical scaffolding* during whole class discussions; where analytic scaffolding is used to "support progress toward the mathematical goals for the discussion" (p. 493). To do this Speer and Wagner used a composite framework drawing on components of Ball et al.'s (2008) *mathematical knowledge for teaching* construct and Shulman's (1986) idea of *pedagogical content knowledge*. They argued that one specific domain of mathematical knowledge for teaching, specialized content knowledge, is needed to support teachers in coming to understand the ideas expressed by students during a discussion, while pedagogical content knowledge is needed to support teachers in determining whether these ideas have the potential to contribute to the mathematical goals of the discussion (and contribute to the students' learning).

We aim to build off the work of Speer and Wagner (2009) by further explicating the role of pedagogical content knowledge in supporting inquiry oriented instruction. Speer and Wagner focused on classroom discussions in which students contributed potential solutions. The mathematician who was leading the discussions struggled to make sense of these solutions and to determine whether they had the potential to move the mathematical conversation forward. In their analyses, Speer and Wagner found that it was the special mathematical work of making sense of the students' contributions that gave the instructor (a research mathematician) the most difficulty.

The teaching episodes we will describe are somewhat different. We will be looking at instances in which students expressed difficulties with the mathematics, whereas Speer and Wagner looked at whole-class discussions in which students shared solutions. As a result, our analyses contribute new insights into the role of teacher knowledge in supporting inquiry-oriented instruction. Additionally, we note that in their analyses Speer and Wagner (2009) used Shulman's (1986) general

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pedagogical content knowledge construct. Ball et al. (2008) refined this construct identifying three sub-domains (knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum). Our research further explicates the role of one of these sub-domains, *knowledge of content and students,* in supporting teachers in the work of identifying and understanding students' mathematical struggles.

1. Theoretical perspective

In an effort to identify specific ways in which mathematical knowledge for teaching (Ball et al., 2008) impacted the implementation of an inquiry-oriented curriculum, we decided to focus on the knowledge needed to support mathematicians as they listened to their students. Davis (1997) discussed three different types of *listening* that a teacher can engage in: hermeneutic, interpretative, and evaluative.

When a teacher engages in hermeneutic listening, the teacher becomes "a participant in the exploration" (Davis, 1997, p. 369). As such, the traditional roles of teacher and student begin to shift, and the class, rather than the teacher, becomes the mathematical authority. Additionally, when the teacher engages in this type of listening, the mode of instruction shifts to be "more a matter of flexible response to ever-changing circumstances than of unyielding progress toward imposed goals" (p. 369). This concept of hermeneutic listening was further built upon by Rasmussen (in Yackel, Stephan, Rasmusen, & Underwood, 2003) with what he called *generative listening*. By engaging in generative listening, the teacher is able to revise the lesson trajectory in response to student contributions. Because of this change in the trajectory of the lesson, generative listening has the ability to "generate or transform one's own mathematical understanding and it can generate a new space of instructional activities" (p. 117). For example, by attending to a student's novel conjecture, an instructor may learn some new mathematics and discover an opportunity to engage the classroom community in an unexpected and productive line of inquiry.

We argue that generative listening requires the teacher to listen to students with the intent of making sense of their thinking. Davis (1997) refers to this type of listening as *interpretative listening*, which he characterizes as listening to students with the aim of "making sense of the sense they are making" (p. 365). Both generative and interpretative listening are contrasted by evaluative listening (Davis, 1997). Evaluative listening is listening with the intent of "trying to simply assess the correctness of the student responses" (p. 365), where the students' responses play no role in determining the trajectory of the lesson.

When analyzing how mathematicians implement inquiry-oriented curriculum, especially curriculum that is heavily influenced by the Realistic Mathematics Education notion of *guided reinvention* (Freudenthal, 1991), we argue that it is of great importance to look at how teachers listen to their students.

> The intent of guided reinvention is that students come to view the mathematics as their own creation. One way for this to occur is for the classroom participants (teachers and students) to lay down a mathematical path as they go, rather than follow a well-trodden trajectory. (Yackel et al., 2003, p. 117)

In order for the classroom participants to be able to create the "mathematical path as they go," student contributions need to shape the trajectory of the lessons. This requires the teacher to listen to the students with the intention of understanding the mathematics being expressed by the students (interpretative listening), and then, when appropriate, leverage the student contributions to inform the trajectory of the lesson (generative listening).

As reported by Speer and Wagner (2009), a mathematician's ability to understand and leverage students' mathematical reasoning in the moment may be heavily reliant on specialized content knowledge (Ball et al., 2008) and pedagogical content knowledge (Shulman, 1986). For instance, Speer and Wagner concluded that, in order to provide analytic scaffolding for whole class discussions, it is necessary to draw on pedagogical content knowledge regarding "typical ways of student thinking" (p. 557). This specific type of pedagogical content knowledge was further elaborated in Ball et al.'s (2008) framework of *mathematical knowledge of teaching*, where knowledge of content and students was defined as "knowledge that combines knowing about students and knowing about mathematics" (p. 401). In addition to knowledge of the mathematical content, "teachers must anticipate what students are likely to think and what they will find confusing" (p. 401).

A primary goal of our research is to build on Speer and Wagner's (2009) observation that pedagogical content knowledge is an important factor for supporting mathematicians as they implement inquiry-oriented curriculum. Specifically, our aim is to further explicate the role of pedagogical content knowledge in listening to students. As we will illustrate, we identify knowledge of content and students as being a particularly important category of pedagogical content knowledge when it comes to the instructor's ability to listen productively to students. Ball et al. (2008) alluded to this relationship when they proposed that knowledge of content and students is drawn upon to "hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language." (p. 401). Additionally, our findings serve to further flesh out the knowledge of content and students category, as our analyses reveal interesting differences among the types of knowledge of content and students that these kinds of listening require.

2. Background: a brief discussion of the curriculum

As part of a curriculum development and research project in abstract algebra, we have been investigating teachers' implementation of an inquiry-oriented abstract algebra curriculum (Larsen, 2009; Larsen, Johnson, Rutherford, & Bartlo,