Journal of Mathematical Behavior 34 (2014) 58–75

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Locally logical mathematics: An emerging teacher honoring both students and mathematics

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ARTICLE INFO

Article history:
Available online 15 March 2014

Keywords:
Teaching practice
Mathematical processes
Prospective secondary mathematics teacher
Teacher knowledge

ABSTRACT

This case study explores the mathematics engagement and teaching practice of a beginning secondary school teacher. The focus is on the mathematical opportunities available to her students (the *classroom mathematics*) and how they relate to the teacher's personal capacity and tendencies for mathematical engagement (her *personal mathematics*). We use a mathematical process-and-action approach to analyze mathematical engagement and then employ the teaching triad—mathematical challenge, sensitivity to students, and management of learning—to situate mathematical engagement within the larger context of teaching practice. The article develops the construct of *locally logical mathematics* to underscore the cogency of mathematical engagement in the classroom as part of a coherent mathematical system that is embedded within a teaching practice. Contributions of the study include the process-and-action approach, especially in tandem with the teaching triad, as a tool to understand nuances of mathematical engagement and differences in demand between written and implemented tasks.

1. Introduction

"*[A] radical function is basically, is an equation with a radical in it that is a function.*" (Mimi, Obs 4 476–478)

Seemingly questionable mathematical claims such as this quote are not atypical of what emerging and practicing teachers state, perhaps stress, in classroom conversations with students. It might be tempting to use such comments as evidence to support claims about how poorly teachers understand mathematics or about the low quality of classroom mathematics experiences. We take an inquisitive stance and ask how such comments make sense within an emerging teacher's understanding of mathematics and view of her classroom context. Understanding the emerging teacher as an apprentice in mathematics teaching who is held responsible for orchestrating mathematical opportunities for students, we look at the teacher's mathematics through a mathematical process-and-action approach. After articulating our framework and approach, we report on the case of a beginning teacher embedded in her classroom contexts as we address the following research question:

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0732-3123/$ – see front matter
http://dx.doi.org/10.1016/j.jmathb.2014.01.003

Table 1
Selected mathematical processes and their descriptions.

Mathematical process	Description of process
Generalizing	Generalizing is the constructive act of extending the domain to which a set of properties applies, either from multiple instances to a class that includes those instances or from a subclass to a larger class of mathematical entities.
Justifying	Justifying (including proving) is the constructive act of explaining how one knows a mathematical claim is true or producing a rationale for belief in a mathematical claim.
Representing	Representing is the constructive act of creating external visuals, physical objects, verbiage or movements intended to capture properties of mathematical entities (e.g., concepts, procedures, principles).
Defining	Defining is the constructive act of identifying and articulating, for a given mathematical entity, a(n ideally minimal) set of mathematical properties or commonalities across a collection of instances of that entity.

> How does a beginning secondary mathematics teacher's personal engagement in mathematical processes and in actions on products of these processes relate to the nature of mathematical processes and actions in her classroom?

We think of *personal mathematics* as the mathematics that an individual knows and how that person engages in mathematics. C*lassroom mathematics* refers to the mathematical content and activities publicly available in a classroom. We pursue an understanding of the relationships between a teacher's personal mathematics and classroom mathematics. Our goal is to extend existing theoretical and empirical work connecting teacher knowledge and teaching practice to provide new insights into how emerging teachers balance student needs, instructional demands, and mathematical engagement as they and their students cocreate mathematical experiences in their classrooms. As we articulate in Section 5, we propose and elaborate *locally logical mathematics* as a construct that captures the mathematical experience in a teacher's classroom and use the construct to explain how statements such as the opening quote make sense within a local context that results from the influence of a school setting and teacher's personal mathematics on her classroom mathematics.

2. Conceptual and theoretical underpinnings

We ground this study in a conceptual frame of classroom and personal mathematical engagement in terms of processes and actions at the content level of the mathematics in the classroom and teaching practice viewed through the Teaching Triad approach (Potari & Jaworski, 2002) at a context level. The content and context levels are blended within a Neo-Vygotskian sociocultural perspective on learning and knowledge (Rogoff, 1990; Wertsch, 1991).

2.1. Classroom mathematics

Classroom mathematics is the mathematical content and activities that are publicly available in a classroom. From Rogoff's (1990) Neo-Vygotskian perspective, classroom mathematics matters because it is the practice of mathematical thinking in which students are apprentices. To understand classroom mathematics as students might experience it, we need to capture how students, teachers, or a combination of students and teacher engage in mathematical processes and actions on the products of those processes.

In our *process-and-action* approach, we focus on four processes—justifying, defining, generalizing and representing (Zbiek, Conner, & Peters, 2008; Zbiek, Heid, & Blume, 2012). These particular processes arose as we, with our colleagues, investigated literature on advanced mathematical thinking and descriptions of the work of mathematicians (e.g., Dreyfus, 1991; Duval, 2007; Goldin, 1998, 2008; Harel & Sowder, 1998; Radford, 2003, 2008; Rasmussen & Zandieh, 2000; Vinner, 1991). Table 1 contains the definitions of these processes that emerged from a synthesis of that literature.

Students and teachers might not only engage in these processes; they might act on justifications, definitions, generalizations, and representations—products that they or others produce. We therefore consider *actions on products*. For example, students might *interpret* a graphical representation produced with a calculator, *apply* a generalization presented by the teacher, or *link* the definition they produce with one that appears in a textbook. We recognize that some actions can be done with all four types of products. For example, students and teachers might *interpret* a representation, definition, generalization, or justification.

In a classroom, the engagement of learners in processes and actions can be complicated and it can be suspended before a product is produced or end in a product that does not fit with conventional mathematics. In fact, learning to experience and move beyond these messy situations is part of students' apprenticeship experience in mathematical thinking. We honor this characteristic of classroom mathematics and analyze situations in which processes and actions are involved, regardless of whether they are carried out to successful or conventional conclusions.