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Understanding the role of transactive reasoning in classroom discourse as students learn to construct proofs☆

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ABSTRACT

This study uses a sociocultural perspective to examine the role of transactive reasoning in, whole-class discourse as undergraduate students learn to construct mathematical proofs. The research, setting is an undergraduate mathematics course with 30 participants. Data are whole-class transcripts, of lessons focused on developing mathematical proofs and students' written assessments on proofs. Transcript data are analyzed for (1) shifts in students' knowledge about proofs; (2) the nature of, transactive reasoning (Berkowitz, Gibbs, & Broughton, 1980) in whole class discourse, including how it occurred and, indications that students appropriated transactive reasoning as a practice of discourse; and (3) how, transactive reasoning supported students' active constructions of proofs and understanding of proof. Results indicate that classroom discourse that helps students appropriate transactive reasoning as a habit of interaction supports their capacity to build arguments about increasingly complex, mathematical ideas and, as such, has positive implications for their learning of proof.

1. Introduction

1.1. Language as a mediator of learning

No utterance is neutral. As Bakhtin noted, "utterances are not indifferent to one another, and are not self-sufficient; they are aware of and mutually reflect one another. . ..[and] every utterance must be regarded primarily as a *response* to preceding utterances" (1986, p. 91). If we accept this claim as an axiom of discourse, it not only points us generally to the reciprocal influence conversation has on its participants, it also raises the particular issue of how teacher utterances in classroom conversation influence student learning. Cazden, noting that classroom discourse happens *among* the students and teacher while the heart of education is to effect learning *within* each student, raises the singular question, "How do the words spoken in classrooms affect this learning" (2001, p. 60)?

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The study of classroom discourse has its roots in language as a mediator of learning. In recent decades, Vygotsky's (1962/1934) sociocultural perspective has generated much interest in the study of how psychological tools—particularly, language—serve to guide human behavior. He argued that "higher voluntary forms of human behavior have their roots in social interaction, in the individual's participation in social behaviors that are *mediated by speech*" [italics added] (Minick, 1996, p. 33) and that language serves as a "manifestation of the transition between social speech on the inter-psychological plane (between individuals) and inner speech on the intra-psychological plane (within the individual)" (Wertsch, 1988, p. 86). Halliday further reasoned that the "distinctive characteristic of human learning is that it is a process of making meaning—a semiotic process; and the prototypical form of human semiotic is language" (1993, p. 93). This dialectic of speech—that is, that language is both tool and result of psychological functioning (Holzman, 1996)—captures the theoretical premise of the study reported here: Learning is shaped by the social context in which it occurs, and the speech occupying that social context encodes a story of development that can be deconstructed through an analysis of discourse.

Prompted by Vygotsky's groundbreaking work, the convergence of thinking embodied in international perspectives on mathematics education (e.g., Wirszup & Streit, 1987) and the rise of social, situated learning perspectives (e.g., Gardner, 1985; Greeno, 1989) has re-cast learning through participation metaphors (e.g., Sfard, 2002). It has also led to calls for educational research that investigates "the relationship between discourse and knowing as it occurs in particular, situated activities" (Wells, 1999, p. 102). All of this has brought what the teacher does—and says—in the mathematics classroom into relief. We are interested here in this dynamic as students learn to construct mathematical proofs in undergraduate classrooms.[2]

1.2. Proof as an essential topic in learning mathematics

The ideas of proving and justifying have received increased attention as a necessary part of students' mathematical development (e.g., Boero, 2007; Committee on the Undergraduate Program in Mathematics, 2004; National Council of Teachers of Mathematics [NCTM], 2000; RAND Mathematics Study Panel, 2002; Stylianou, Blanton & Knuth, 2009). Most recently, the Common Core State Standards Initiative [CCSSI] (2010) identified constructing viable arguments as one of its eight core Mathematical Practices. Indeed, claims that the "essence of mathematics lies in proofs" (Ross, 1998, p. 2) and that proof is "the soul of mathematics" (Schoenfeld, 2009, p. 12) reinforce the centrality of proof in mathematical thinking.

In spite of this, students' difficulty in learning how to understand and construct proofs is well documented (e.g., Senk, 1985; Stylianou, Blanton, & Rotou, 2014; Usiskin, 1987). Studies have found, for example, that students struggle with what constitutes a proof and with understanding the power of a generalized argument as covering all possible cases (e.g., Fischbein & Kedem, 1982; Healy & Hoyles, 2000). Moreover, they have difficulty with the logic and methods of proof (e.g., Duval, 1991) and the problem solving skills necessary to construct arguments (Schoenfeld, 1985).

While studies such as these give us critical information in understanding how one learns to read and construct proofs, they focus on individual cognitive processes. Yet, in light of contemporary ways of knowing, studies are also needed that examine the social aspects of teaching and learning proof (Alibert & Thomas, 1991). In response to this, the research reported here is an effort to understand the role of whole-class discourse, specifically, the form of teacher and student utterances, in the development of undergraduate students' learning of proof.[3]

2. Constructs constituting our frameworks for analyzing students' learning of proof

In this section, we discuss the constructs constituting our frameworks for analyzing students' learning of proof and use these constructs to pose the research questions that frame this study.

2.1. Using transactive reasoning to analyze classroom discourse

We found *transactive reasoning*, defined by Berkowitz, Gibbs, and Broughton (1980, presented in Kruger, 1993) as criticisms, explanations, justifications, clarifications, and elaborations of one's own or another's ideas, to be a useful construct for analyzing whole-class discourse. Applying transactive reasoning to a study that sought to characterize the mechanism of change in peer discussions on socio-moral dilemmas, Kruger (1993) found that children's transactive reasoning about solutions they challenged and eventually rejected was closely associated with positive posttest performance. Kruger argued that as children critiqued each other's thoughts in the transactive discussion of rejected solutions, they co-constructed understanding. As such, dissent and conflict became vehicles for change in thinking. Elsewhere, Goos, Galbraith, and Renshaw (2002) focused on how students used transactive reasoning in mathematical problem solving in small group peer discussions. Similar to Kruger, they found that transactive discussions were a significant source of productive metacognitive activity because these discussions led to the public scrutiny of ideas among peers. Further, Goos et al. (2002) claim that this metacognitive activity due to transactive reasoning led to successful problem solving; students not only engaged in desirable behaviors (such as checking their work due to criticisms of others), but also increased their performance.

[2] We take "mathematical proof" here to refer to a logical, deductive argument constructed within the accepted standards of the mathematics community.

[3] We take the term "discourse" here to refer to verbal utterances.