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Teachers' mathematical activity in inquiry-oriented instruction

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ABSTRACT

This work investigates the relationship between teachers' mathematical activity and the mathematical activity of their students. By analyzing the classroom video data of mathematicians implementing an inquiry-oriented abstract algebra curriculum I was able to identify a variety of ways in which teachers engaged in mathematical activity in response to the mathematical activity of their students. Further, my analysis considered the interactions between teachers' mathematical activity and the mathematical activity of their students. This analysis suggests that teachers' mathematical activity can play a significant role in supporting students' mathematical development, in that it has the potential to both support students' mathematical activity and influence the mathematical discourse of the classroom community.

Teachers' successful implementation of inquiry-oriented curricula requires a focus on students – both on their ways of understanding and their mathematical practices. This is especially true with curriculum based on the Realistic Mathematics Education (RME) notion of *guided reinvention* (Freudenthal, 1991), in which "the idea is to allow learners to come to regard the knowledge they acquire as their own private knowledge, knowledge for which they themselves are responsible" (Gravemeijer & Doorman, 1999, p. 116). One way in which RME based curricula promote such ownership is by promoting the evolution of formal mathematics from students' informal understandings (Gravemeijer, 1999). In this way, the concepts first emerge from the students' informal activity and then develop into more formal ways of reasoning. The teacher has a crucial role in this transition. Namely, the teacher needs to finds ways to build on the students' informal ideas in order to help them construct the formal mathematics. As a result, teachers implementing such curricula must be active participants in establishing the mathematical path of the classroom community while at the same time allowing students to retain ownership of the mathematics. The goal of this research is to better understand what it is that teachers *do* while teaching in order to achieve these instructional goals.

The research presented in this paper is part of a larger research agenda designed to understand the challenges and opportunities that emerge as different faculty implement an RME-based, inquiry-oriented abstract algebra curriculum. In order to develop instructor support materials for the *Teaching Abstract Algebra for Understanding* (TAAFU) curriculum (Lockwood, Johnson, & Larsen, 2013), it became important to (1) identify the mathematical activities that teachers implementing the TAAFU curriculum engage in during classroom teaching in response to the mathematical activity of their students, and (2) investigate the ways in which teachers' mathematical activity interacts with students' mathematical activity. This paper will address the complexity of teaching with an RME based, inquiry-oriented curriculum by analyzing teachers' mathematical activity as they work to support the mathematical activity of their students. To this end, classroom video data from two

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implementations of an inquiry-oriented abstract algebra course were analyzed to understand the kinds of mathematical activity that teachers engaged in when responding to and supporting the mathematical activity of their students.

1. Theoretical perspective

RME was developed to be consistent with the idea that, "mathematics education should take its point of departure primarily in *mathematics as an activity*" (Gravemeijer & Doorman, 1999, p. 116). This view of mathematics, elaborated by Freudenthal (1991) below, rejects the view of mathematics as a ready-made-system to be memorized by students.

> Mathematics as an activity is a point of view quite distinct from mathematics as printed in books and imprinted in minds. . .That is mathematics as an activity of discovery and organising in an interplay of content and form (p. 16–17)

Consistent with this perspective of mathematics as an activity, the research literature describes a number of student mathematical activities. For instance, Rasmussen, Zandieh, King, and Teppo's (2005) *advancing mathematical activity* includes symbolizing, algorithmatizing, and defining as specific examples of mathematical activity. Further, in order to understand and generate mathematical proofs, students would likely engage in proof related activities, such as evaluating arguments (Selden & Selden, 2008), instantiating concepts (Weber & Alcock, 2004), and proof analysis (Larsen & Zandieh, 2007). Still other mathematical activity that students are likely to engage in as they work to reinvent mathematical concepts includes conjecturing, questioning, and generalizing.

As students engage in such mathematical activity, one would expect that teachers would need to engage in mathematical activity in response. For instance when faced with a novel student-generated proof a teacher may need to evaluate the proof to determine the validity of the argument and possible advantages/disadvantages of this new approach, both in terms of the current task and in terms of the goals for the lesson. As part of evaluating a student's proof, the teacher may engage in proof analysis (Larsen & Zandieh, 2007), such as searching for hidden assumptions. Further, the teacher may need to identify connections between the student's proof technique and other mathematical justifications the students would be likely to encounter later in the course (Johnson & Larsen, 2012).

The research literature on teachers' implementations of reform curriculum provides a number of examples of mathematical work done by teachers in response to their students' mathematical activity. At the undergraduate level, Speer and Wagner (2009) presented a study in which they sought to account for the difficulties a mathematician was facing while trying to provided analytic scaffolding during whole class discussions, where analytic scaffolding is used to "support progress towards the mathematical goals of the discussion" (p. 493). Speer and Wagner identified several skills necessary for providing analytic scaffolding, including the ability to recognize and figure out both the ideas expressed by their students and the potential for these ideas to contribute to the mathematical goals of the lesson. Speer and Wagner went on to state that, "recognizing draws heavily on a teacher's PCK (pedagogical content knowledge), whereas *figuring out requires that a teacher do some mathematical work in the moment* [emphasis added]" (p. 8).

Johnson and Larsen (2012) investigated a mathematician's ability to interpretively and/or generatively listen to her students' contributions. Interpretive listening involves a teacher's intent to make sense of student contributions and generative listening reflects a readiness for using student contributions to generate new mathematical understanding or instructional activities (Davis, 1997; Yackel, Stephan, Rasmusen, & Underwood, 2003). In order to engage in interpretive and/or generative listening, a mathematician may need to interpret a student's imprecise language, generalize a student's statement into a testable mathematical conjecture, or identify counterexamples to a student's claim (see Johnson & Larsen, 2012), all of which require mathematical work on the part of the teacher. Therefore, interpretive and generative listening are examples of teaching practices that are supported by a teacher's mathematical work.

Examples of teaching practices that are likely to require mathematical work are not limited to research on mathematicians teaching undergraduate mathematics. Studies focused on in-service and pre-service elementary teachers have also identified analyzing student work, interpreting student explanations, and building on student contributions as important instructional activities needed for teaching mathematics (Charalambous, 2008, 2010; Hill et al., 2008). Each of these tasks requires teachers to engage in mathematical activity in response to the mathematical activity of their students. Fig. 1 lists some of the kinds of mathematical activity that have been identified in the research literature as activities that teachers engage in during classroom teaching in response to the mathematical activity of their students.

It is important to note that in all of these examples, the teachers' mathematical activity is (1) in response to student mathematical activity, and (2) connected to pedagogical considerations, such as advancing the mathematical agenda or assessing student work. Given the context for this mathematical work (teaching), it seems likely that a teacher's mathematical activity may support students' mathematical activity indirectly in the sense that teachers' mathematical activity would inform their *pedagogical activity*. For instance, providing counterexamples, stating the formal mathematical version of a student contribution for a class discussion, and exhibiting a proof for the class could all be examples of pedagogical activity that was informed by a teacher's mathematical activity.

In each of these examples the teacher's pedagogical activity introduces new mathematics into the classroom discourse. The teacher's contribution serves to alter, test, refine, or expand the mathematical ideas under development. Thus, pedagogical activity that introduces new mathematics into the classroom discourse is likely an indicator of teacher mathematical activity. Such teacher activity and corresponding mathematical contributions are of particular interest because, once introduced into the mathematical discourse of the classroom community, these contributions have the potential to support