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Spatial structuring and the development of number sense: A case study of young children working with blocks

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ABSTRACT

This case study discusses an activity that makes up one of five lessons in an ongoing classroom teaching experiment. The goal of the teaching experiment is (a) to gain insight into kindergartners' spatial structuring abilities, and (b) to design an educational setting that can support kindergartners in becoming aware of spatial structures and in learning to apply spatial structuring as a means to abbreviate and ultimately elucidate numerical procedures. This paper documents children's spatial structuring of three-dimensional block constructions and the teacher's role in guiding the children's learning processes. The episodes have contributed to developing the activity into a lesson that could foster children's use of spatial structure for determining the number of blocks. The observations complement existing research that relates spatial structuring to mathematical performance, with additional insight into the development of number sense of particularly young children in a regular classroom setting.

1. Introduction

What repeatedly stands out from studies on early childhood development is the apparently natural drive of young children to go out and explore the world. In fact, researchers have drawn parallels between children, scientists and poets with regard to their sense of wonder and the intense way in which they experience the world (Gopnik, 2004; Gopnik, Meltzoff, & Kuhl, 1999). What is disconcerting, then, is that "early childhood education, in both formal and informal settings, may not be helping all children maximize their cognitive capacities" (National Research Council, 2005). Moreover, several researchers have warned about the apparent gap between children's informal, intuitive knowledge and interests, and the formal learning opportunities at the start of their schooling (Clements & Sarama, 2007; Griffin & Case, 1997; Hughes, 1986). This is reflected in many early elementary mathematics curricula that recognize the importance of number sense (Casey, 2004; Clements & Battista, 1992) without accrediting the spatial sense that children typically develop at a very early age (Ness & Farenga, 2007; Newcombe & Huttenlocher, 2000).

An overwhelming body of research has discussed the development of mathematical thinking in terms of either spatial sense or number sense. Relatively few studies, however, have considered what role early spatial sense may play in supporting the development of number sense. Such an association seems viable in light of studies that have related elementary students' spatial structuring abilities to their counting skills (Battista & Clements, 1996; Battista, Clements, Arnoff, Battista, & Van Auken Borrow, 1998) and early school mathematical performance (Mulligan, Prescott, & Mitchelmore, 2004; Mulligan,

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Mitchelmore, & Prescott, 2005; Mulligan, Mitchelmore, & Prescott, 2006). Research on how to support the development of number sense of particularly kindergarten children (aged 4–6 years) is important because around that age (a) children are building a more solid foundation for their number sense (Griffin, 2004a), while, at the same time, (b) they are expected to bridge their relatively intuitive and informal mathematical knowledge with the more complex mathematics of a formal school setting (Clements & Sarama, 2007).

This "revolutionary" period in the development of mathematical thinking (Griffin, 2004a) has inspired the present ongoing investigation into the development of and association between early spatial sense and emerging number sense of kindergarten children. The aim of this research is to gain a more in-depth understanding of how the development of young children's number sense may best be supported in an educational setting (Van Benthem, Dijkgraaf, & De Lange, 2005; Van Nes & De Lange, 2007; Van Nes & Doorman, 2006). For this purpose, classroom activities were developed that are meant to encourage young children's use of flexible numerical procedures using their early spatial sense, and help them prepare for formal school mathematics.

In this paper, we present a case study in the form of one particular classroom activity that involves the counting of blocks in a three-dimensional construction. This activity is one of five lessons that were conducted in a classroom teaching experiment to investigate the development of kindergartners' spatial structuring abilities. The purpose of this paper, then, is to explore the lesson with regard to (a) kindergartners' spatial structuring ability and (b) the characteristics of an educational setting (i.e., the teacher and the instruction activity) that may support the development of spatial structuring abilities in three-dimensional settings. These abilities are expected to help the children to eventually gain insight into numerical relations that may help them to learn to abbreviate numerical procedures such as determining, comparing and operating with quantities.

After investigating the theoretical association between spatial and number sense, we document the children's responses to the activity as well as the proactive role of the teachers. In light of current theories on the role of spatial structuring for supporting the development of number sense, this should encourage the implementation of such an activity in a kindergarten classroom for supporting the development of children's insight into numerical relations.

2. Theoretical background

2.1. Defining number sense

Number sense can broadly be defined as the ease and flexibility with which children operate with numbers (Gersten & Chard, 1999). In summarizing an extensive list of components, Berch (2005) stated that

> possessing number sense ostensibly permits one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex math problems; from making simple magnitude comparisons to inventing procedures for conducting numerical operations; and from recognizing gross numerical errors to using quantitative methods for communicating, processing, and interpreting information (p. 334)

Early quantitative abilities include children's ability to *subitize* and compare quantities by making correspondences (Clements & Sarama, 2007; Van den Heuvel-Panhuizen, 2001). Subitizing may be defined as an automated perceptual process that all people can apply only to small collections of up to around four objects (see also *perceptual subitizing*, Clements, 1999). As children progress in their ability to count, they discover easier ways of operating with numbers. They come to understand that numbers can have different representations and can act as different points of reference (Berch, 2005; Griffin & Case, 1997; Van den Heuvel-Panhuizen, 2001). The present research specifically focuses on awareness of quantities, on giving meaning to quantities and on relating the different meanings to each other (Van den Heuvel-Panhuizen, 2001), because such knowledge is necessary for determining a quantity (i.e., counting), for comparing quantities and for preliminary arithmetic. This requires insight into *numerical relations* which can be achieved through the structuring (e.g., splitting or decomposing and composing) of quantities (Hunting, 2003; Steffe, Cobb, & Von Glasersfeld, 1988). Such insight, in turn, underlies a well-founded number sense and determines the ease with which children progress to an understanding of higher order mathematical skills and concepts (Griffin, 2004b; Van den Heuvel-Panhuizen, 2001).

2.2. Defining spatial sense

Spatial sense encompasses the ability to "grasp the external world" (Freudenthal, in National Council of Teachers of Mathematics [NCTM], 1989, p. 48). Within the vast body of research that exists on spatial sense, the three main components of spatial sense that appear most essential for "grasping the world" and for developing mathematical thinking are spatial visualization, spatial orientation, and shape (cf. Clements & Sarama, 2007; Owens, 1999).

Spatial visualization involves the ability to mentally picture the movements of two- and three-dimensional spatial objects. In spatial visualization tasks, all or part of a representation may be mentally moved or altered (Bishop, 1980; Clements, 2004; Tartre, 1990a), requiring object-based transformations where the frame of reference of the observer stays fixed (Zacks, Mires, Tversky, & Hazeltine, 2000). Young children already apply spatial visualization skills, for example, when they imagine where in the kitchen they can find a snack.

The second component of spatial sense that we study is spatial orientation. This is what Clements (2004, p. 284) refers to in describing how we "make our way" in space. In spatial orientation, the self-to-object representational system is at