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Issues in theorizing mathematics learning and teaching: A contrast between learning through activity and DNR research programs

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ABSTRACT

By continuing a contrast with the DNR research program, begun in Harel and Koichu (2010), I discuss several important issues with respect to teaching and learning mathematics that have emerged from our research program which studies learning that occurs through students' mathematical activity and indicate issues of complementarity between DNR and our research program. I make distinctions about what we mean by inquiring into the mechanisms of conceptual learning and how it differs from work that elucidates steps in the development of a mathematical concept. I argue that the construct of disequilibrium is neither necessary nor sufficient to explain mathematics conceptual learning. I describe an emerging approach to instruction aimed at particular mathematical understandings that fosters reinvention of mathematical concepts without depending on students' success solving novel problems.

1. Introduction

What might it mean to study mathematics conceptual learning? What is the potential of research on conceptual learning for informing mathematics teaching and curriculum development? I explore these questions using a contrast between two research programs.

Harel and Koichu (2010) provided an "operational definition of learning" mathematics. In that article, they made eight references to one of our recent publications (Simon et al., 2010), using our work as a point of comparison and contrast. As I read Harel and Koichu's article, it became clear to me that the comparison and contrast would serve well to raise some additional issues related to mathematics learning and teaching.[1] Examples of counterpoint can lead to communication and clarification of ideas.

Harel and Koichu build on Harel's DNR theoretical framework (2008a, 2008b). The DNR framework is one of the most elaborated frameworks for problem-solving classroom lessons. In using the contrast with DNR, my goal is to highlight a complementary approach to conceptualizing mathematics learning and teaching. The emerging conceptualization of learning and teaching also provides the basis for a type of instructional design that is *not* grounded in problem solving. Harel (2013) explained that *problem solving* "is usually defined as engagement in a problem 'for which the solution method is not known in advance"' (National Council of Teachers of Mathematics, 2000, p. 52). Problem solving is an essential and important aspect of mathematical activity and mathematics instruction. The complementary instructional design approach that we

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[1] I thank Guershon Harel for stimulating conversations based on drafts of this article.

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put forth does not stress ways to help students learn to wrestle productively with novel problems. Rather it is meant to be another arrow in the instructional design quiver. It does not replace problem-solving lessons. Its purpose is to promote conceptual learning for intractable concepts and students struggling to master particular concepts. Because this work is in an early stage, we have not yet developed broad classroom models of integrating this approach with problem solving lessons.

In discussing this complementary approach to research on mathematics learning and teaching, I focus on three related issues:

1. Inquiry into the mechanisms of mathematics conceptual learning,
2. The role of the construct of disequilibrium in theorizing mathematics learning and teaching, and
3. A theoretical basis for teaching particular mathematical understandings.

This discussion is focused primarily on the characterizations of learning found in Harel and Koichu (2010) and Simon et al. (2010), even though these characterizations derived from earlier work by the lead researchers and their colleagues.[2] I present only elements of the DNR framework and research program that are useful points of contrast. Readers are encouraged to get a fuller picture from the primary sources referenced in this article. The contrast that I offer in this article is not primarily about conflicting ideas, but rather about different goals and methodology. In some cases, the contrast might point to complementarity. Before engaging in this discussion, I wish to set the context by making three points.

1. Learning is a complex and varied process. Studying learning can be like the traditional Indian story of the "Blind Men and the Elephant." In the story, each blind man reported different characteristics, because of the part of the elephant that he was exploring with his hands. The story can be interpreted as men quarreling out of ignorance, because they each had only partial information. Studying learning is more difficult than coming to know an elephant with one's hands, because learning can only be explored inferentially. However, as an analogy to studying learning, we can take the Indian parable one step further. That is, if the blind men (think "researchers") know that they are only exploring a part of their object of study, collective progress can be made by considering each characterization in relation to each man's (researcher's) relative position (their goals, interests, theoretical frameworks, and methodologies). My aim in this paper is to use the contrast between these two bodies of work to clarify ideas relative to particular goals, constructs, and research activities with which my collaborators and I have studied and continue to study mathematics learning.[3]
2. There are several theories of learning in use in mathematics education. The two bodies of work discussed in this paper build on elements of some of those theories. It is not my purpose in this paper to provide an overview of theories of learning. Both research groups in prior reports have related their theoretical work to extant literature. Rather I focus narrowly on a small set of ideas that can be highlighted through contrasting our work with that of Harel and Koichu.
3. Most of Harel's research over the last dozen years, which provided the context for his development of the DNR framework, has been focused on advanced mathematics.[4] In contrast, my own research has focused mostly on the mathematics of elementary and middle school. Although it is beyond the scope of this article, future work might elucidate whether particular characterizations of mathematics learning may be more useful for different mathematical levels.

2. D–N–R

Harel and Koichu's (2010) article is based on Harel's (2008a, 2008b) conceptual framework, called *DNR-based instruction in mathematics*. The DNR theoretical framework is a comprehensive framework focused on the learning and teaching of mathematics. The initials D, N, and R stand for three instructional principles central to the framework: *duality*, *necessity*, and *repeated reasoning*. Harel (2008b) defined these principals as follows:

> The Duality Principle: Students develop ways of thinking through the production of ways of understanding, and, conversely, the ways of understanding they produce are impacted by the ways of thinking they possess (p. 899).
> The Necessity Principle: For students to learn the mathematics we intend to teach them, they must have a need for it, where 'need' here refers to intellectual need (p. 900).
> The Repeated Reasoning Principle: Students must practice reasoning in order to internalize desirable ways of understanding and ways of thinking (p. 900).

[2] Our earlier work was done in collaboration with Prof. Ron Tzur (see Simon et al., 2004; Simon & Tzur, 2004; Tzur & Simon, 2004).

[3] The characterization of the DNR work used for this contrast makes use of articles in print. There is no attempt to draw conclusions about where the DNR work will go in the future or the interests of the researchers involved.

[4] Prior to that time, he was involved in extensive research on the teaching and learning of rational numbers.