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A geometry teacher's use of a metaphor in relation to a prototypical image to help students remember a set of theorems

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ABSTRACT

This article asks the following: *How does a teacher use a metaphor in relation to a prototypical image to help students remember a set of theorems?* This question is analyzed through the case of a geometry teacher. The analysis uses Duval's work on the *apprehension of diagrams* to investigate how the teacher used a metaphor to remind students about the heuristics involved when applying a set of theorems during a problem-based lesson. The findings show that the teacher used the metaphor to help students recall the apprehensions of diagrams when applying several theorems. The metaphor was instrumental for mediating students' work on a problem and the proof of a new theorem. The findings suggest that teachers' use of metaphors in relation to prototypical images may facilitate how they organize students' knowledge for later retrieval.

1. Introduction

Prior work has shown that students use metaphors to apply their knowledge of everyday life to their understanding of mathematical ideas (Presmeg, 1998). Additionally, metaphors help students learn how to communicate mathematical ideas (Lakoff & Núñez, 2000; Pimm, 1981, 1988; Presmeg, 1997, 1998). Pimm (1988) provided the following useful perspective regarding metaphors: "Metaphor involves the seeing (and therefore the understanding) of one thing in terms of another; it is a conceptual rather than solely a linguistic phenomenon" (p. 30). The use of metaphors facilitates the generation of new ways of speaking about mathematical concepts in relation to other known concepts (Pimm, 1988). Moreover, metaphors can help students develop new mathematical meanings by establishing new connections with non-mathematical ideas (Lakoff & Núñez, 2000, p. 379).

In this paper, I ask the following: *How does a teacher use a metaphor in relation to a prototypical image to help students remember mathematical content that was previously taught in class?* I present the case of a geometry teacher's use of a metaphor during a problem-based lesson that was taught in two classes. I use this case as a descriptive study illustrating teachers' use of metaphors to trigger students' memories of specific heuristics when applying a set of theorems. This case provides an example of a teacher's effort to organize students' knowledge such that the students can later retrieve that knowledge by building didactic memories (Brousseau & Centeno, 1991; Flückiger, 2005). Didactic memory consists of the concepts, mathematical propositions (e.g., theorems, axioms, and definitions), and procedures that a teacher officially introduced in

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class. By building students' didactic memory, a teacher can keep track of the mathematical ideas that, from the teacher's perspective, students should know.

Examining cases within mathematics education is particularly important in light of current mathematics education reform, which promotes problem-based instruction (National Council of Teachers of Mathematics [NCTM], 2000, p. 52). Researchers have demonstrated that problem-based instruction is challenging given that it increases teachers' attention to students' work as a means of introducing students to the concepts and procedures that are the target of a lesson (Chazan, 2000; Herbst, 2006; Lampert, 2001; Lubienski, 2000). In a problem-based lesson, teachers strive to "allow mathematics to be problematic" for students while simultaneously supporting the students' mathematical learning (Hiebert & Wearne, 2003, p. 13). Ambitious instructional strategies, such as problem-based instruction, require a deep understanding of teachers' capabilities and current work (Cohen, 2011; Cohen & Moffitt, 2009). A better understanding of teaching practices when engaging students in problem-based instruction could inform researchers and teacher educators about how reform-based curriculum can most efficiently be implemented.

Ultimately, this case aims to investigate elements of the *practical rationality of mathematics teaching* (Herbst & Chazan, 2003, 2011; Herbst, Nachlieli, & Chazan, 2011) through an analysis of instructional strategies. The practical rationality of mathematics teaching involves teachers' ways of coping with teaching problems through their performance of actions that are deemed reasonable from their perspective. The examination of specific instructional strategies may highlight the apparent elements of the practical rationality of mathematics teaching, as these strategies may seem unusual to an ordinary observer, but make sense to a mathematics teacher. In addition to identifying examples of teachers' instructional practices, I present possible justifications that teachers may have for implementing such strategies.

I use the teachers' actions and, when available, the teachers' commentaries about their actions as evidence for identifying the elements of the practical rationality of mathematics. That is, teachers' identification of the problems with regard to teaching, such as the issue of organizing students' memories, and the manner in which they attempt to address these problems are my sources for understanding the actions that make sense from a practitioners' perspective. In accordance with Herbst et al. (2011), I also assume that many elements of the practical rationality of mathematics teaching are tacit (Cook & Brown, 1999). Therefore, classroom observations of the instances in which teachers have to address practical problems while teaching are crucial for identifying viable solutions to these problems from the teachers' perspective.

2. Research questions

The following research questions guided this case study:

1. What is the relationship between the use of a metaphor and a prototypical image?
2. What apprehensions of a diagram elicit the use of a prototypical image that is associated with a metaphor?

These two questions intend to describe how a teacher uses a metaphor to activate students' memories of theorems through the relationship between a prototypical image and a metaphor.

3. Theoretical framework

The theoretical framework informing this work includes broad perspectives about collective remembering through conversations and specific perspectives about the creation of a didactic memory. This work provides the background for investigating the influence of metaphors and prototypical diagrams when working with geometric diagrams in a geometry class.

3.1. Collective remembering in classrooms

The notion that individuals who belong to a social group share memories has been a topic of discussion in different domains, such as sociology and history (e.g., Anastasio, Ehrenberger, Watson, & Zhang, 2012; Douglas, 1986; Halbwachs, 1992; Matsuda, 1996; Ricoeur, 2004). This notion is supported by the assumption that there is something greater than an individual memory and that memory is distributed socially among members of a group, which expands on the traditional use of the psychological notion of memory (Manier, 2004). Recently, Anastasio and colleagues (2012, pp. 2–3) established an analogy between individual and collective memory processes by using the term "consolidation" to describe changes in the status of some information from being transitory to more stable. With regard to collective memory, the consolidation process occurs with the support of structures that help organize the information that is remembered by members of a group. Researchers who examine social interactions are focusing on the process of creating and consolidating memories, rather than on the actual memories that a group possesses. Wertsch (2002) proposed the notion of *collective remembering* to describe the dynamic process through which a group creates and appropriates shared memories of the past through resources that help mediate the development of such memories. Prior work has documented how conversations are important for understanding the process of collective remembering (Edwards & Middleton, 1988; Middleton & Brown, 2005; Middleton & Edwards, 1990; Middleton, 1997) given that speakers use specific strategies to support the goal of remembering during conversations (Goodwin, 1987; Harris, Keil, Sutton, Barnier, & McIlwain, 2011). Prior work has also proposed that artifacts,