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An exploration of students' errors in derivatives in a university of technology

Sibawu Witness Siyepu*

Cape Peninsula University of Technology, South Africa

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ABSTRACT

This paper reports on an exploration of errors that were displayed by students who studied mathematics in chemical engineering in derivatives of various functions such as algebraic, exponential, logarithmic and trigonometric functions. The participants of this study were a group of twenty students who were at risk in an extended curriculum programme in a university of technology in Western Cape, South Africa. The researcher used a qualitative case study approach and collected data from students' written work. This research uses action, process, object, and schema (APOS) theory to classify errors into categories and to analyse and interpret the data collected. The students displayed five different kinds of errors, namely, conceptual, interpretation, linear extrapolation, procedural and arbitrary. The use of APOS theory as a framework revealed that several students' errors might be caused by over-generalisation of mathematical rules and properties such as the power rule of differentiation and distributive property in manipulation of algebraic expressions. This study suggests that teaching of the standard rules of differentiation should put emphasis on its restrictions to eliminate common errors that normally crop up due to over-generalisation of certain differentiation rules.

1. Introduction and rationale for the study

Globally there is a high failure rate among first year university mathematics students (Moru, 2006; Naidoo & Naidoo, 2007). This is attributed to various reasons such as students who enter universities already at-risk. At-risk students are students who are not experiencing success in their schooling system. They are usually low academic achievers who show low confidence. The identification of at risk students and the development of programmes to prevent their failure are necessary components of educational reform in universities (Margarita, 1987).

Engelbrecht, Harding, and Phiri (2010) assert that "several lecturers who taught first-year mathematics in 2009 reported on under-preparedness of students" (p. 4). Padayachee, Boshoff, Olivier, and Harding (2011) support this, as they explain that "from the experience of teaching first-year mathematics students, they noticed that many first-year students are under-prepared for mathematics" (p. 1). Engelbrecht et al. (2010) further explain that this challenge faced by universities need the curriculum responsiveness. To assist the students who are at risk to improve their performance, a university of technology (UOT) in the Western Cape, South Africa admits these students in an extended curriculum programme (ECP).

ECP is a curriculum that is designed for students who are borderline cases. These students do not meet the minimum academic requirements for admission to the main engineering stream, but show potential based on psychometric testing to succeed in their studies. The minimum requirement for admission in the main engineering stream is that students should

* Tel.: +27 021 460 3335/073 285 2609; fax: +27 021 460 3711.
E-mail address: siyepus@cput.ac.za

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at least obtain 50% in mathematics, physical science and English in the matriculation examination, as well as entrance to a university of technology.

The UOT also established a mathematics support programme where the researcher is employed as a mathematics lecturer. One of the researcher's job descriptions is to assist students who are at risk to improve their performance in the learning of mathematics. In an extended curriculum programme, students study the same content of mathematics as other students in the main engineering stream, but instead of completing it within a semester; they have to do it over a period of one year. The way this is done is to add active learning components such as group work, projects, peer work and other related support work such as how to read the subject texts and how to solve problems and represent knowledge in the field (Siyepu, 2010, pp. 241–242).

The current ECP mathematics syllabus is dominated by calculus with a large component of differentiation that consists of derivatives of algebraic, exponential, logarithmic and trigonometric functions. This is a new section that students come across for the first time in their first-year level of study in universities. However, there are few studies reported on exploration of errors in derivatives of algebraic, exponential, logarithmic and trigonometric functions.

The essence of this study is to explore students' errors in derivatives of algebraic, exponential, logarithmic and trigonometric functions in order to develop techniques of addressing the errors that are displayed by the students who participated in this study.

2. Literature review

Students' poor performance in mathematics is alarming, particularly in differentiation (Siyepu, 2011). The difficulties that are encountered by students when learning mathematics are seen in errors that they display in their solutions. These errors have been discussed and documented by various researchers such as Brodie (2005, 2006, 2010); Davis (1984); Drews (2005); Foster (2007); Hatano (1996); Luneta and Makonye (2010); Nesher (1987); Olivier (1989); Orton (1983); Ryan and Williams (2000) and Smith, DiSessa, and Rosehelle (1993).

2.1. The nature of errors

The nature of errors is based on mistakes displayed by students when they attempt to solve mathematical problems. Students demonstrate different mistakes, which arise owing to many different reasons. Researchers such as Nesher (1987); Olivier (1989) and Smith et al. (1993) categorise these mistakes as either slips, errors or misconceptions. Olivier (1989) distinguishes between slips, errors and misconceptions as follows:

- Slips are wrong answers owing to processing; they are not systematic, but are carelessly made by both experts and novices; they are easily detected and are quickly corrected;
- Errors are wrong answers owing to planning; they are systematic in that they are applied regularly in the same circumstances. Errors are the symptoms of the underlying conceptual structures that are the cause of errors; and
- Underlying beliefs and principles in the cognitive structure that are the causes of systematic conceptual errors are known as misconceptions (p. 3).

A misconception is a conceptual structure, constructed by the learner, which makes sense in relation to her/his current knowledge, which is not aligned with conventional mathematical knowledge (Confrey, 1990; Nesher, 1987; Smith et al., 1993). In this study the researcher uses the word "errors" to refer to systematic, persistent and pervasive patterns of mistakes performed by students across a range of contexts (Nesher, 1987).

Luneta and Makonye (2010) highlight that although errors and misconceptions are related, they are different. They define an error as a "mistake, slip, blunder or inaccuracy and a deviation from accuracy" (p. 35). Riccomini (2005) states that unsystematic errors are unintended, non-recurring wrong answers which students can readily correct by themselves. Systematic errors, though, are recurrent wrong responses, methodologically constructed, and produced across space and time. Researchers such as Green, Piel, and Flowers (2008); Nesher (1987); and Riccomini (2005) explain that systematic errors are symptomatic of a faulty line of thinking referred to as misconceptions. Errors arise from students' prior learning, either in the mathematics classroom or from their interaction with the physical and social world (Smith et al., 1993). They claim that misconceptions are intuitively sensible to students and can be resilient to instruction designed to correct them (Smith et al., 1993). Errors are visible in students' artefacts such as written text or speech. However misconceptions can even be hidden in correct answers (Smith et al., 1993), when correct answers are accidental. Other researchers such as Davis (1984) and Olivier (1989) claim that errors arise from an over-generalisation of a concept from one domain to another.

2.2. Generalisation

Mathematics uses patterns in generalisation to develop students' understanding of algebra. Mason (1996) claims that "a lesson without the opportunity for students to generalise is not a mathematics lesson" (p. 65). Learning mathematics