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Justification as a teaching and learning practice: Its (potential) multifacted role in middle grades mathematics classrooms

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ABSTRACT

Justification is a core mathematics practice. Although the purposes of justification in the mathematician community have been studied extensively, we know relatively little about its role in K-12 classrooms. This paper documents the range of purposes identified by 12 middle grades teachers who were working actively to incorporate justification into their classrooms and compares this set of purposes with those documented in the research mathematician community. Results indicate that the teachers viewed justification as a powerful practice to accomplish a range of valued classroom teaching and learning functions. Some of these purposes overlapped with the purposes in the mathematician community; others were unique to the classroom community. Perhaps surprisingly, absent was the role of justification in verifying mathematical results. An analysis of the relationship between the purposes documented in the mathematics classroom community and the research mathematician community highlights how these differences may reflect the distinct goals and professional activities of the two communities. Implications for mathematics education and teacher development are discussed.

Justification is a practice at the heart of mathematics. As a disciplinary practice, justification has many purposes: it is used to validate claims, illuminate or provide insight into a result or phenomenon, and systematize knowledge, among others (Bell, 1976; de Villiers, 1990, 1999, 2002; Hanna, 1990, 2000). We know much less about the role of justification in K-12 classrooms (when it is present). Justification may be used in classrooms for purposes similar to those of mathematicians, but it may also play a role in other classroom-relevant aims or purposes (Knuth, 2002b; Staples & Truxaw, 2009).

Our focus on justification is not derived exclusively from its import as a disciplinary practice, but also from its role as a *learning practice* (Cohen & Ball, 2001). As a learning practice, justification is a *means by which* students enhance their understanding of mathematics and their proficiency at doing mathematics; it is a means to learn and do mathematics. There is empirical support for this connection as students in classrooms where they are prompted for their mathematical rationales express more complex and higher levels of mathematical thinking (Hiebert et al., 1997; Wood, Williams, & McNeal, 2006) and demonstrate greater student learning outcomes (Boaler, 1997; Kazemi & Stipek, 2001). Furthermore, classrooms that engage students in justification may support more equitable outcomes among heterogeneously grouped, diverse populations (Boaler, 2006; Boaler & Staples, 2008).

Currently, there is a general dearth of justification in US mathematics classrooms (Jacobs et al., 2006), even when teachers are implementing proof-related tasks (Bieda, 2010). This state of affairs is clearly a cause for concern. A deeper and more nuanced understanding of the practice of justification in K-12 mathematics classroom communities is critical if we are to expand the presence of this practice and meet the current visions of reform documents which increasingly emphasize the

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importance of justification (e.g., Common Core State Standards [CCSS], 2010; National Council of Teachers of Mathematics [NCTM], 2000). Specifically, we need to better understand how teachers think about justification in the context of their work. Teachers are deliberate actors, and their conceptions of the subject matter, aims and purposes, can play an important role in shaping their classroom practices (Fennema & Franke, 1992; Knuth, 2002a, 2002b).

In this paper, we explore the purposes of justification in middle grades classrooms. We first discuss justification and the potential relationship between the practice of justification in the mathematics classroom community and the research mathematician community. Next, we review literature on what is known about the purposes of justification in each community – the classroom community and research mathematician community. We then report findings from our study with a group of 12 middle school teachers, highlighting places of confluence and divergence between the purposes identified by the teachers and the purposes documented in the literature about the research mathematician community. Finally, we explore factors that contribute to the overlap and uniqueness of each set and discuss implications of these findings. For ease in communication, we subsequently use the term *mathematician community* to indicate the *research mathematician community*, and specify when we intend other communities of mathematicians.

In addition to documenting teachers' perspectives, we compare the purposes of justification of these mathematics teachers with the purposes documented for research mathematicians for two reasons. First, a sizable set of research on justification in classrooms uses the purposes of the mathematician community as a frame of reference (e.g., de Villiers, 1999; Hanna, 2000; Knuth, 2002b). Consequently, this framing provides us the opportunity to contribute to this body of literature. Second, it is important to document when valued purposes or practices of a disciplinary community are and are not a vibrant aspect of the classroom communities in which that subject is taught. Such documentation is the first and necessary step toward making sense of why these similarities and differences may exist, evaluating if these differences are appropriate or desirable, and subsequently making adjustments to practice and/or our conception of alignment between communities. There is a need to make connections between these two communities of practice, and at the same time, to acknowledge that they serve different purposes which may shape their justification practices.

1. Justification and proof

Every discipline (and community) has its own standards for what counts as justification and, correspondingly, what is required to establish a conjecture or theory as a (working) truth. For example, in science, a strong empirical foundation is needed, as well as coherence with the prevailing scientific theories (which have been established based on empirical evidence) (Kuhn, 1962). In mathematics, establishing a new result generally requires a rigorous deductive argument, presented following agreed upon conventions, that demonstrates the truth of a mathematical claim, that is, a proof.

Theorists and researchers have no single, agreed upon definition of proof (CadwalladerOlsker, 2011; Jaffe, 1997), or of related terms such as informal proof and justification, although some argue that there is a fair amount of agreement on the definition of formal proof (Balacheff, 2002). For the purposes of this inquiry, we have chosen to use the term justification and define justification as *an argument that demonstrates (or refutes) the truth of a claim that uses accepted statements and mathematical forms of reasoning*. Our definition of justification is similar to Stylianides' (2007) definition of arguments that function as proofs in a given classroom community. It differs in that we do not focus on the community aspect, nor do we try to make judgments about what forms of reasoning and representations are within the conceptual reach of the community engaged in the proving process.

Note that the type of reasoning used in the argument must be a *mathematical form* of reasoning (Larsen, McCaffrey, & Staples, in preparation; Larsen et al., 2011). Consequently, this definition excludes arguments that use reasons such as "because John told me" or "because that's what you said was right yesterday." These are not *mathematical* forms of reasoning. The definition, however, does permit arguments such as "That's what we proved yesterday" which is an appeal to previously established results, and is a mathematical form of reasoning and used consistently in mathematics. It also permits empirical or example-based reasoning as a mathematical form of reasoning. Although in many instances, an empirical argument will not demonstrate the truth of a claim, this form of reasoning can be used to prove a claim depending on the question. For example, if students are asked to find the perimeter of the 10th and 50th figures in a pattern, an empirical argument, where the student counts, is valid.

Both the process of justifying and also the end point of having constructed a justification are relevant for thinking about the purposes and value of justification in the classroom. The same ideas play out when discussing proof and proving. As CadwalladerOlsker (2011) describes, "Proving is a process, which may include arguments and trains of thought which ultimately lead nowhere. The proof, which is the result of this process, will not include such dead ends" (p. 39). The value of proof, however, encompasses both. For example, a documented role of proof is discovery, but that discovery can come about through the process of proving – with its dead ends and false starts which evolves into a new discovery.

Our notion of justification is consistent with the Common Core's (CCSS, 2010) description of the key practice *Construct viable arguments and critique the reasoning of others.* The document states the following:

> Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. (pp. 6–7)