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Geometry students' hedged statements and their self-regulation of mathematics

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ABSTRACT

Statements conveying a degree of certainty or doubt, in the form of hedging, have been linked with logical inference in students' talk (Rowland, 2000). Considering the current emphasis on increasing student autonomy for effective mathematical discourse, I posit a relationship between hedging and student autonomy. In the current study, high school Geometry students' frequency of producing hedged mathematical statements were correlated with their perceived mathematical autonomy to determine if a relationship existed. Results found a strong and statistically significant correlation, providing support for a connection between students' hedging and their perceived autonomy. However, additional analysis revealed that perceptions of mathematical competence and social relatedness were also influential to hedging. Implications of these results are discussed.

1. Introduction

Expression of statements indicative of logical inference have the potential for shaping discussion in mathematics classrooms and deepening the meaning-making students engage in during mathematical discussions. One manner in which logical inference surfaces in mathematical dialog is through the usage of hedged statements (Polya, 1954; Rowland, 2000). Inherent in such statements is the expression of degrees of certainty or doubt in a student's mathematical talk (e.g., "I *think* the angle bisector *might* bisect the opposite side" versus "the angle bisector bisects the opposite side"). Halliday and Matthiessen (2004) argue that use of such chance-like language is indicative of an interpersonal function within the grammar itself; thus providing other speakers more opportunities to respond to such statements. The combination of such attributes suggests that hedged statements may be one of many characteristics of effective mathematical discussion; where 'effective' suggests such dialog allows for the deepening of mathematical knowledge. Therefore, a pragmatic goal for mathematics teachers and mathematics educators is to examine ways in which we can facilitate student production of hedged statements in their mathematical talk.

One potential means of such facilitation is to increase the sense of mathematical autonomy students' posses in mathematics classrooms. Several authors focusing on mathematical discussion suggest that the degree of autonomy a student has, or is provided to have, contributes to their engagement in mathematical discussion (e.g., Krummheuer, 2007; Lo, Whetly, & Smith, 1994; Yackel & Cobb, 1996). Further, certain authors (Krummheuer, 2007; Rowland, 1999) make connections between student autonomy and logical inference. Therefore, it seems appropriate to examine the relationship between student perceived mathematical autonomy and their production of hedged statements. A useful lens for such an examination is Self-Determination Theory (SDT), which posits that for an individual to be self-regulated within a context they must have

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a fulfilled sense of autonomy in that context (Deci, Vallerand, Pelletier, & Ryan, 1991; Ryan & Deci, 2000). However, Ryan and Deci (2000) also suggest other malleable factors influence student autonomy; competence and social relatedness.

The primary purpose of this study is to investigate whether a relationship exists between Geometry students' perceived mathematical autonomy and their production of hedged statements. The results of such analysis will help clarify whether allowing for a certain measure of student control in the mathematics classroom has a relationship with whether such students speak in any way indicative of logic; as pertains to the forms of logic expressed through hedging. Within the following pages, I provide an overview of forms of logical inference and their expression in mathematical dialog as well as a description of the grammatical elements of one form of conveyed logical inference (i.e., hedging). These descriptions are followed by a discussion of SDT in mathematical discourse and an overview of the theoretical framework.

1.1. Logic in mathematical discussion

Peirce (1982) distinguishes logical reasoning into three forms: abduction, induction and deduction. Deduction is the reasoning of a result deduced from a rule. As Peirce (1982, 1992b) describes it, one considers a rule, then a particular case applicable to the rule, and finally produces a result based on the rule and the case. Induction, however, begins with a particular case, followed by a result and then produces a rule. Where a rule is the starting point of deduction, it is the product or end point for induction. Induction involves generalization from a case or number of cases. Abduction (sometimes referred to as conjecture or hypothesis) is a lesser form of inference, as described by Peirce (1992b). It is ". . .where we find some very curious circumstance, which would be explained by the supposition that it was a case of a certain general rule, and thereupon adopt that supposition" (Peirce, 1982, p. 326). The degree of inference (deductive, inductive or abductive) is based upon our evidence for it (Peirce, 1982). Peirce argued that an inference is based on information present or known, and as such, different inferences will have different likelihoods of truth. A weak inference will have a lower chance of being able to be found true, not based on the merits of the argument but on the evidence it is based upon.

Abduction designates something as possible (Paavola, 2005), while induction has more precision than abduction (Peirce, 1992b). Arguably combining abduction with induction, Polya (1954) argued that people reason inductively about mathematics based on the innate probability and reasonableness of their claims. Polya suggested that through simple uses of words such as *likely* and *probable*, individuals convey a sense of probability, and make judgments based on it, even though the sense of probability conveyed is vague (Polya, 1954). Rowland (2000) cites Polya's linkage of induction and chance and combines it with Peirce's conceptions of abduction and induction. In discussing vagueness in mathematical discourse, Rowland shows this element of chance inherent in induction, as well as abduction, is present in students' spoken conjectures. Specifically, Rowland described that the vagueness in students' spoken descriptions alluded to abductive and inductive logic.

While Rowland (2000) did not address deduction in his examination of vagueness in students' descriptions, Peirce (1992b) did assign a degree of chance to deduction. Deduction possesses the probability of certainty. "A definite probability always attaches to the Deductive conclusion because the mode of inference is necessary" (p. 141). Therefore, by indicating something *must* be or *certainly* is, an individual may be using deductive inference.

In summation, the hierarchy that Peirce (1992b) organized for his triad of reasoning is related to degrees of certainty and these degrees of certainty appear to convey themselves in how people talk about mathematics (Polya, 1954; Rowland, 2000). Abduction conveys the least amount of certainty in the three logical inferences with induction being a stronger form than abduction. Deduction is a strong logical inference with 'definite probability.' So, a statement spoken with an inherent aspect of certainty can be considered to have some association with some form of logical inference (e.g., the angles *might* be congruent; the angles *probably* are congruent; the angles *must* be congruent). Therefore, I refer to such statements as *hedged statements*, adopting the terminology used by Lakoff (1973), Rowland (2000), and others. While evidence supports a connection between a hedged statement as conveying logical inference (e.g., Polya, 1954; Rowland, 2000), the converse is not true. The student statement "So that means [the angle] *has to* equal 140" is a deductive inference with hedging, yet the student could have said "so that means [the angle] equals 140," producing a polar deductive inference (one without hedging); a result based on a rule. Similarly, induction and abduction might be conveyed without use of hedging.

2. Semantic properties of hedged statements

Systemic Functional Linguistics (SFL) examines language through its grammatical properties in the context of the meaning that is conveyed through its use. Halliday and Matthiessen (2004) outline three metafunctions of grammar: ideational, interpersonal, and textual. I focus here on the interpersonal metafunction. The interpersonal metafunction is concerned with "enacting our personal and social relationships" (Halliday & Matthiessen, 2004, p. 29) through language. As part of the interpersonal metafunction, the systems of polarity and modality are linguistic resources for a speaker to convey their certainty, or not, of conveyed information.

Polarity allows a speaker to convey statements as spoken-as-fact (Halliday & Matthiessen, 2004) regardless of whether the statement is actual fact. For example, in one class discussion about angle relations, a student stated "this is a linear pair," which is spoken-as-fact. While another participant in the discussion may have chosen to provide a detailed response, the statement itself invites a limited range of responses. Yet, if the student had included a modal in their statement so that they stated "this *might* be a linear pair," the statement invites much more in the way of responses. Whereas polarity conveys a statement as spoken-as-fact, modality provides a contrast in that it conveys a degree of certainty or uncertainty (Halliday