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Rationals and decimals as required in the school curriculum Part 4: Problem solving, composed mappings and division

Guy Brousseau[a], Nadine Brousseau[b], Virginia Warfield[c,*]

[a] University of Bordeaux, Bordeaux, France
[b] École Michelet, Bordeaux, France
[c] Department of Mathematics, University of Washington, Box 354350, Seattle, WA 98195-4350, USA

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ABSTRACT

In the late seventies, Guy Brousseau set himself the goal of verifying experimentally a theory he had been building up for a number of years. The theory, consistent with what was later named (nonradical) constructivism, was that children, in suitable carefully arranged circumstances, can build their own knowledge of mathematics. The experiment, carried out by a team of researchers and teachers that included his wife, Nadine, in classrooms at the École Jules Michelet, was to teach all of the material on rational and decimal numbers required by the national program with a carefully structured, tightly woven and interdependent sequence of "situations." This article describes and discusses the fourth and last portion of that experiment.

1. Introduction

This article is the fourth and last of a series presenting the sequence of 65 lessons of a curriculum devoted to an experimental study of the teaching of rational and decimal numbers during compulsory education (common to all citizens). Conceived in 1972 and launched as an experiment in 1973, this curriculum was not a project intended for distribution and reproduction in the system at large, but a design for experimental study of certain questions of didactics.[1]

In the three preceding articles we described the first 10 modules of the experiment. Together with the last modules (14 and 15) they were reproduced almost identically every year between 1974 and 1997 in two parallel fifth grade classes at the Michelet School in Talence. They can thus be recounted after 10 years of testing as the same sequence of unchanging lessons, simply enriched by some specific observations.[2] Here we complete the cycle and demonstrate its mathematical coherence.

Modules 12 and 13 were not introduced into the curriculum in question until 1984–1985. They were brought about by results from earlier research (on mathematical situations) and were the instrument for new research (on didactical situations in mathematics). A study of the paradoxes of the untenable didactical contract demonstrated in 1981 that the actions of the teachers cannot be independent of the knowledge taught (Brousseau, 1996). The construction of a new result in mathematics is a personal process, independent of environment. There may be collective effort involved, but the process itself is internal. It cannot be the same for a student in a didactical environment. In order to give to the student the opportunity to construct

* Corresponding author. Tel.: +1 206 525 0175.
E-mail address: warfield@math.washington.edu (V. Warfield).

[1] Published in various articles between 1972 and 1986, in particular in *Recherches en Didactique des Mathématiques* 1981 and 1982 and in Brousseau (1995).

[2] "Rationals and decimals in...", a 500-page manuscript distributed by the IREM of Bordeaux in French-speaking centers of research in mathematics education (Brousseau & Brousseau, 1985).

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personal thinking (by problems), an effort is necessary to suspend the flow of external information. But subsequently, it is necessary to recognize and reform this thinking to put it in conformity with current knowledge. It was therefore necessary to study the teaching of problem solving in everyday contexts, which had previously been left at the discretion of the teachers. The added modules bear witness to this work in progress. They made use of examples to address the teachers and suggest didactical means for modifying the students' relationship with problems, but they never led to formally constituted lessons and reproducible observations as the others did. They are still under study today, but we decided nonetheless to present them in the order set up in the manual, before modules 14 and 15.

Finally, we cannot ignore the challenge of presenting to an American reader today, even clarified, such a quantity of teaching activities and of conclusions plucked out of their technical, historical, geographic, scientific and conceptual context. With the aim of preventing misunderstandings, we have set aside the last section of this article for an attempt to answer some of the questions that must arise for the reader: Why should he be interested in this curriculum? It wasn't set up to be developed.[3] It cannot even be reproduced in today's cultural, economic and social environment.[4] It refers to an exotic and revolutionary epistemological context. It partially replaces the direct study of the behaviors of students by that of the situations that cause the behaviors.

This experiment and certain other similar ones[5] are central in the emergence of Didactics of Mathematics. But this holistic research is very ambitious because its intention is to demonstrate simultaneously the inappropriacy of numerous widely accepted beliefs. Furthermore, it relies on a sophisticated and apparently somewhat unorthodox methodology.[6] A discussion of all of these reasons would be beyond the scope of an article, so that we are only able to mention certain aspects. As a foundation both for the discussion we have included within the article and for future discussions, we present the facts of the experiment itself – how it was carried out, and what was observed about it. We ask only that the reader withhold judgment about the things that justify and explain our actions until after reading the notes and commentary that constitute the last sections of the article.

What follows consists of three types of texts. Two are from the original text (Brousseau & Brousseau, 1985), which we will refer to as GNB1985): (1) class episodes (interventions of the teacher and the students) and (2) commentary designed to help the teachers understand and reproduce the lessons from one year to another. Third are the 2008 commentaries addressed to the readers of this article.

2. The presentation of module 11 and the supplementary modules 12 and 13: linear problems and the study of them (a 2008 commentary)

From 1973 to 1983 the teachers at the Michelet School completed the modules of the curriculum with a classical exploration of the rest of the official program – the metric system and applications – organized on the habitual model. This part let us reassure ourselves that our new introduction of mathematical notions did not have a negative impact on the performances of the students. Having established this, we were able to respond to the request of the teachers to study a common organization of the problem sessions.

At the end of the 1970s research on mathematical situations such as those we present here had brought to light first the paradoxes of the *didactical contract*[7] and then the necessity of determining the didactical role of the teacher by modeling the whole *didactical situation* and not only the mathematical situation that it contains. Indeed, the actions of the teacher themselves vary – must vary – according to the mathematical notion being studied.

Whatever the methods, teachers must "represent" the infinite set of possible elementary arithmetic problems by a collection of "typical problems" that is limited, but generates the rest. Then they replicate these types in ordinary problems so that the most important of them are revisited sufficiently often. How should these types be determined?

The initial organization of the set (module 11) left the choice of classic problems to the teachers. At their request we decided in 1981 to include a "problem study" in the curriculum under experimentation.

(a) First we studied the problems whose solution is copied from manipulations that can actually be realized, particularly for division (module 12).
(b) After that it was a matter of extending this form of classification and correcting certain inconveniences in the organization of the set in order:
  - to adjoin new problems to the old one,
  - to have the class first use a small number of mathematical terms as a reference for thereafter translating the older terminology and approaches, and of using, at least implicitly, new "organizations" of the set of problems based on
    - o mathematical conceptions (measures, ratios, linear functions),
    - o well known obstacles and difficulties (division by a fraction, unaccustomed numerical values, etc.)

[3] Many parts of the experimental design are explicitly contraindicated for this use: commensuration, arrows, etc.
[4] It was not possible to set up a second COREM, although that was indispensible.
[5] For example, the articles published by JMB: The Case of Gaël, Probability and Stats, etc. (ref.).
[6] Guy Brousseau, "l'observation des activités didactiques", Revus Française de Pédagogie no. 45 1978.
[7] cf. Brousseau & Warfield, 1999.