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Learning beginning algebra with spreadsheets in a computer intensive environment

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ABSTRACT

This study is part of a large research and development project aimed at observing, describing and analyzing the learning processes of two seventh grade classes during a yearlong beginning algebra course in a computer intensive environment (CIE). The environment includes carefully designed algebra learning materials with a functional approach, and provides students with unconstrained freedom to use (or not use) computerized tools during the learning process at all times. This paper focuses on the qualitative and quantitative analyses of students' work on one problem, which serves as a window through which we learn about the ways students worked on problems throughout the year. The analyses reveal the nature of students' mathematical activity, and how such activity is related to both the instrumental views of the computerized tools that students develop and their freedom to use them. We describe and analyze the variety of approaches to symbolic generalizations, syntactic rules and equation solving and the many solution strategies pursued successfully by the students. On that basis, we discuss the strengths of the learning environment and the open questions and dilemmas it poses.

1. Introduction

During the last five years we have conducted a long-term research study whose overarching goal was to observe, describe and analyze the mathematical activity of students in a beginning algebra course within a computer intensive environment (CIE).

We define a CIE as a learning environment in which (1) computerized tools are available to learners at all times (both in class and at home) and (2) the learners are free to choose if, when, and how to use the tools in their work on problem situations. At the beginning of the course, the teacher used the learning materials to introduce the students to the computerized tools and suggested how these may be used to solve certain problems. However, shortly thereafter, none of the problems (which are at the core of the activity in this CIE and were designed to provide ample opportunities to work with computerized tools) included any instructions or specific requests to work either with paper and pencil or with computerized tools. Thus the intensiveness of the environment refers to the *availability* of the computerized tools at all times and not necessarily to an intensive use of them by the students.

The yearlong beginning algebra course designed and implemented for this CIE is organized around a sequence of problem situations around phenomena involving changing quantities. In this "functional approach", equations consist of pointwise comparisons within a changing phenomenon (e.g. Heid, 1995; Hershkowitz et al., 2002; Yerushalmy & Schwartz, 1993), and

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syntactic algebraic skills are integrated into the learning process when needed and at the service of the mathematical activity related to the problem situations.

Our long-term research includes different components described elsewhere: the design principles of the learning environment, its implementation and results of a comparison with student achievements in other classes (Tabach, Arcavi, & Hershkowitz, in press; Tabach, Hershkowitz, Arcavi, & Dreyfus, in press); analysis of student learning of syntactic aspects of algebra (Tabach & Friedlander, 2008); understanding the concept of equation in the beginning algebra course in the CIE (Tabach & Friedlander, 2006), and the students' evolving uses of spreadsheets throughout the course (Tabach, Arcavi et al., in press; Tabach, Hershkowitz et al., in press).

In this paper, we focus on the ways students (from two different seventh grade classes throughout 2 consecutive school years) worked on a problem situation within the CIE. The work of the students in this problem situation serves as a window to examine, characterize and analyze (both quantitatively and qualitatively), the kinds of mathematical activity which took place in this CIE for beginning algebra.

This study touches upon three confluent areas, and it is aimed as a contribution to each of them: learning in computer intensive environments, learning beginning algebra with a functional approach using spreadsheets (e.g. Tabach, Hershkowitz, & Schwarz, 2006), and processes of instrumental genesis in which computerized tools affect thoughtful users (e.g. Vérillon & Rabardel, 1995). In the next section, we briefly discuss these three areas.

2. Background

2.1. Learning in a computer intensive environment

In the last decade, there is a growing interest in learning environments in which computers are available to students and teachers at all times. Studies on these environments usually focus on outcomes, showing advantages and gains, such as improvement of reading and writing skills, better organization of written work as a whole (especially argumentation capabilities), improvement of self-esteem, involvement and the like (e.g. Gardner, Morrison, & Jarman, 1993; Rockman et al., 1997, 1998, 1999). Whereas there are reports of partial uses of computers in mathematics classrooms or labs, there are almost no reports on the teaching of mathematics in a CIE, where computerized tools are available at all times. Moreover, in many cases, students who had had experience in a general CIE, report that mathematics is the subject in which the computer use is the lowest (Rockman et al., 1998), and problem solving usually takes place with pencil and paper only, even when the computer is fully available (Rockman et al., 1997). Lewis (2005) reports on some of the problems she, as a mathematics teacher, faces when coming to implement a CIE in her class because of the absence of suitable learning materials. The scarcity of reports on mathematics in CIE is surprising, in light of the growing body of research on teaching and learning mathematics in computerized environments, where the computer use is partial (a lesson or a sequence of several lessons). One of the goals of this study is to illustrate and discuss the function of a CIE in a beginning algebra classroom with learning materials designed ad hoc—where the computerized tools (mainly spreadsheets) are available to the students at all times.

In the last decade and a half, several algebra projects based on the partial use of different kinds of computerized tools were developed, implemented and studied (e.g. Dettori, Garuti, & Lemut, 2001; Haspekian, 2005; Hershkowitz et al., 2002; Kieran, 1992; Tabach et al., 2006; Wilson, Ainley, & Bills, 2005; Yerushalmy & Schwartz, 1993). Some studies result in questioning the integration of computerized tools in the algebra classroom. In particular, the potential of students using spreadsheets to express algebraic relationships in the form of equations is challenged (e.g. Dettori et al., 2001). Also, there are concerns regarding the kind of mathematics students engage in, in computerized learning environments (e.g. Hershkowitz & Kieran, 2001), and/or with the shift of difficulties from one area to another (e.g. Yerushalmy, 2005).

Other studies emphasize the contributions of computerized tools to the (student) learning of algebra, and their potential to address not only the syntactic aspects but also to focus on (student) understanding in addition to the development of symbol sense (e.g. Arcavi, 1994) and on mathematical modeling. Furthermore, the claim is that the functional approach – which lends itself so well to computerized environments – presents students with different kinds of changing phenomena and opportunities for generalization and modeling within several representations (e.g. Bednarz, Kieran, & Lee, 1996; Yerushalmy, 2005). Such aspects of algebra can be learned by making use of graphical, numerical and symbolic representations whose static nature with paper and pencil become dynamic in computerized environments. Students can choose a graphical, numerical or symbolic representation, or use them in parallel according to their needs and/or personal preferences. These various representations might be used to contrast and operate upon mathematical objects, thus, as Kaput (1992) states, they turn *display* notation systems in a pencil and paper environment into *action* notation systems in computerized environments. Changing representations can be observed, initiated, and reflected upon becoming the sources of investigations and insight.

The use of computerized environments in algebra may also amplify student's capabilities and significantly change the nature of the mathematical activity itself (Pea, 1985). Explorations with computerized tools encourage students to plan, reflect, produce explanations and engage in classroom discussions (Heid, 1995).

Appropriate and successful uses of technological tools in beginning algebra classes have been described, for example, with explorations of every day life problem situations using several representations (e.g. Heid, 1995; Hershkowitz et al., 2002), with numerical experimentation which evolves into functional connections (e.g. Kieran, 1992), and with manipulations of symbolic and graphical representation of functions (e.g. Tabach et al., 2006; Yerushalmy & Schwartz, 1993).