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An inquiry-oriented approach to undergraduate mathematics

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Abstract

To improve undergraduate mathematics learning, teachers need to recognize and value characteristics of classroom learning environments that contribute to powerful student learning. The broad goal of this special issue is to share such characteristics and the theoretical and empirical grounding for an innovative approach in differential equations called the Inquiry Oriented Differential Equations (IO-DE) project. We use the IO-DE project as a case example of how undergraduate mathematics can build on theoretical and instructional advances initiated at the K-12 level to create and sustain learning environments for powerful student learning at the undergraduate level. In addition to providing an overview of the five articles in this special issue, we highlight the theoretical background for the IO-DE project and provide a summary of two quantitative studies done to assess the effectiveness of the IO-DE project on student learning.

Keywords: Inquiry-oriented instruction; Instructional intervention; Realistic mathematics education; Differential equations

A significant number of university mathematics departments, faced with the combination of an increasingly diverse student body, a declining number of mathematics majors, and looming accountability concerns, are more aware than ever of their need to develop effective and innovative curricula and instructional practices (Bok, 2005; Holton, 2001; U.S. Department of Education, 2006). These innovations need to be capable of supporting students in developing deep conceptual understandings of important mathematical ideas as well as productive dispositions. How to accomplish this daunting task is an open question that offers an opportunity for mathematicians and mathematics educators to work together on problems of teaching and learning.

The Inquiry Oriented Differential Equations (IO-DE) project is such a collaborative effort. It seeks to explore the prospects and possibilities for improving undergraduate mathematics education, using differential equations as a case example. In addition to providing an overview of the articles in this issue, the goals of this article are to highlight the theoretical background for the IO-DE project and to provide a summary of two quantitative studies done to assess the effectiveness of the IO-DE project on student learning.

1. IO-DE background theory

Perhaps not surprisingly, different research communities characterize inquiry in different ways. For example, in science education the National Research Council (1996) states that inquiry includes identification of assumptions, use of

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critical and logical thinking, and consideration of alternative explanations. In the philosophy of mathematics education, Richards (1991) characterizes inquiry as learning to speak and act mathematically by participating in mathematical discussions, posing conjectures, and solving new or unfamiliar problems. Both characterizations highlight important aspects of student activity. While such characterizations of student activity are essential, they only address part of the process of inquiry. In order to more fully understand the complexity of classroom learning, our definition of inquiry also encompasses *teacher* activity as well as student activity. In particular, IO-DE teachers routinely *inquire* into their students' mathematical thinking and reasoning. Teacher inquiry into student thinking serves three important functions. First, it enables teachers to construct models for how their students interpret and generate mathematical ideas. Second, it provides opportunities for teachers to learn something new about particular mathematical ideas, in light of student thinking. Third, it better positions teachers to build on students' thinking by posing new questions and tasks.

IO-DE students, on the other hand, learn new mathematics through *inquiry* by engaging in mathematical discussions, posing and following up on conjectures, explaining and justifying their thinking, and solving novel problems. Thus, the first function that student inquiry serves is to enable students to learn new mathematics through engagement in genuine argumentation. The second function that student inquiry serves is to empower learners to see themselves as capable of reinventing mathematics and to see mathematics itself as a human activity.

Mathematically, the IO-DE project drew its initial curricular inspiration from contemporary, dynamical systems approaches, such as those developed by Blanchard, Devaney, and Hall (1998) and Hubbard and West (1991). These approaches represent a significant departure from conventional treatments that emphasize a host of analytic techniques for solving special classes of well-posed problems. Stepping away from the conventional approach, Blanchard, Devaney, and Hall, for example, develop analytic, graphical, and numerical approaches as three distinct methods for analyzing solutions to differential equations. They also incorporate application problems to illustrate the usefulness of differential equations in real world problems.

As our extended research team[1] systematically investigated the learning and teaching in such approaches, we developed three goals for inquiry-oriented learning and teaching that extend contemporary dynamical systems approaches.

- First, we wanted students to essentially reinvent many of the key mathematical ideas and methods for analyzing solutions to differential equations. Thus, rather than being introduced to analytic, graphical, and numerical methods as pre-existing, IO-DE students are invited to engage in challenging problems that provide an opportunity for them to create their own analytical, graphical, and numerical approaches. Teachers, for their part, facilitate and support the growth of students' self-generated mathematical ideas and inscriptions, often toward more conventional ones. The three functions of teacher inquiry support this goal.
- Second, and related to the first goal, we wanted challenging tasks, often situated in realistic situations, to serve as the starting point for students' mathematical inquiry. These tasks would stand in stark contrast to application type problems that typically appear at the end of a section. In other words, *experientially real*[2] situations should drive the need for, and creation of, key mathematical ideas that lead to various methods of solving differential equations. Teachers' careful attention to student thinking has helped us identify such experientially real situations.
- Third, we wanted a balanced treatment of analytic, numerical, and graphical approaches, but we wanted these various approaches to emerge more or less simultaneously for learners, rather than as three isolated, disparate methods. The desire for co-emergence of methods stemmed, in part, from earlier research conducted in a reform-oriented differential equations course in which the students exhibited very compartmentalized understandings of analytic, numerical, and graphical methods (Rasmussen, 1997, 2001). For us, a teacher's responsibility to assist students to identify analytical, numerical, and graphical methods as three different techniques comes only *after* the students have made significant progress in reinventing such methods.

[1] The extended research team includes Mark Burtch, Mi-Kyung Ju, Karen Allen Keene, Michael Keynes, Karen King, Oh Nam Kwon, Karen Marrongelle, Bernd Rossa, Wei Ruan, Kyunghee Shin, Michelle Stephan, Joe Wagner, and Erna Yackel.

[2] The term *experientially real* refers to problem situations for which learners can engage their existing ways of reasoning to make progress on the problem. Such experientially real situations may be grounded in real world settings, but depending on the background and experience of learners, may also be grounded in more symbolic, mathematically oriented settings.